S 100 3 and 4

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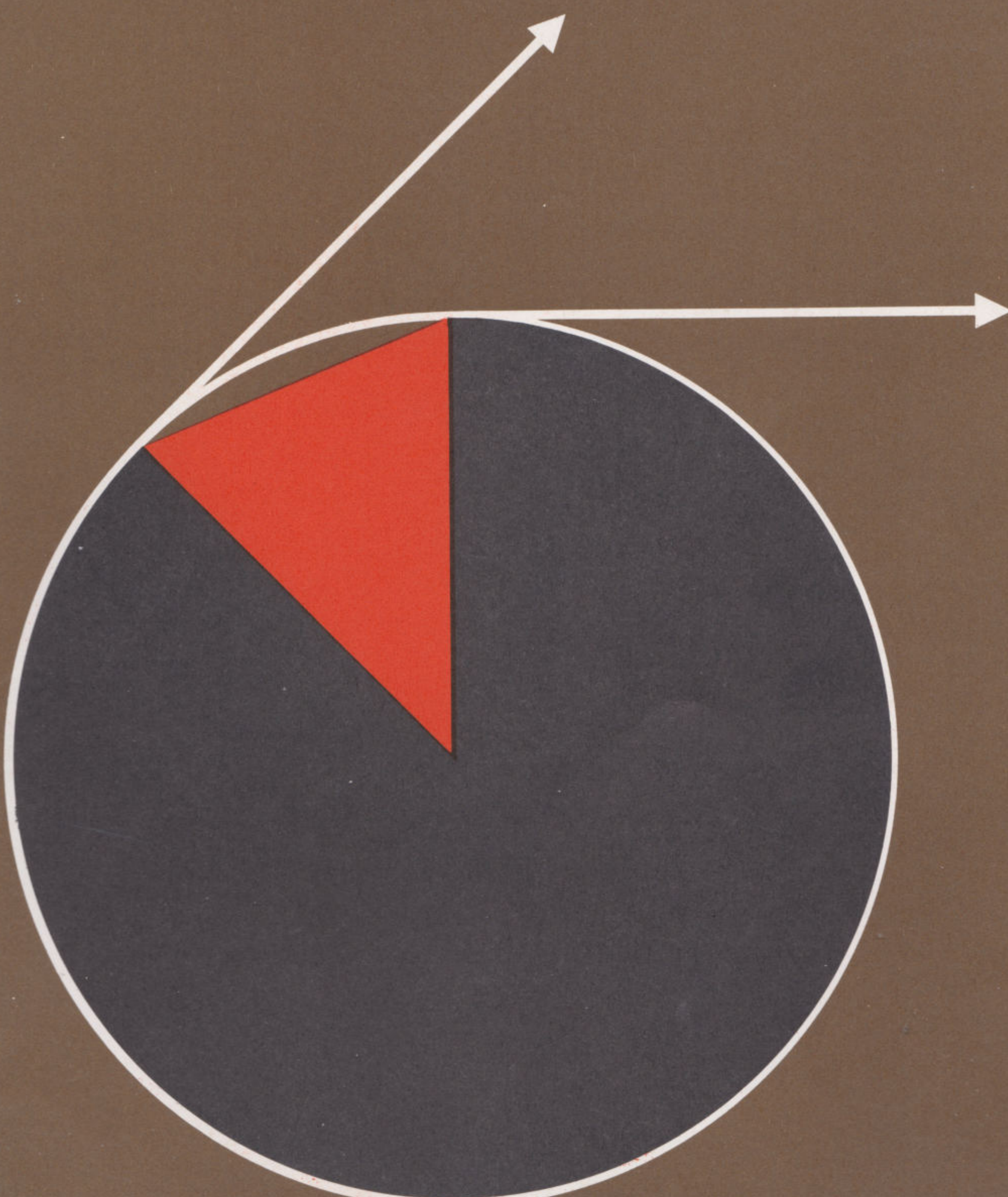
Science Foundation Course Units 3 and 4

Mass, Length and Time
Forces, Fields and Energy



The Open University

Science Foundation Course Unit 4

FORCES, FIELDS AND ENERGY

Prepared by the Science Foundation Course Team

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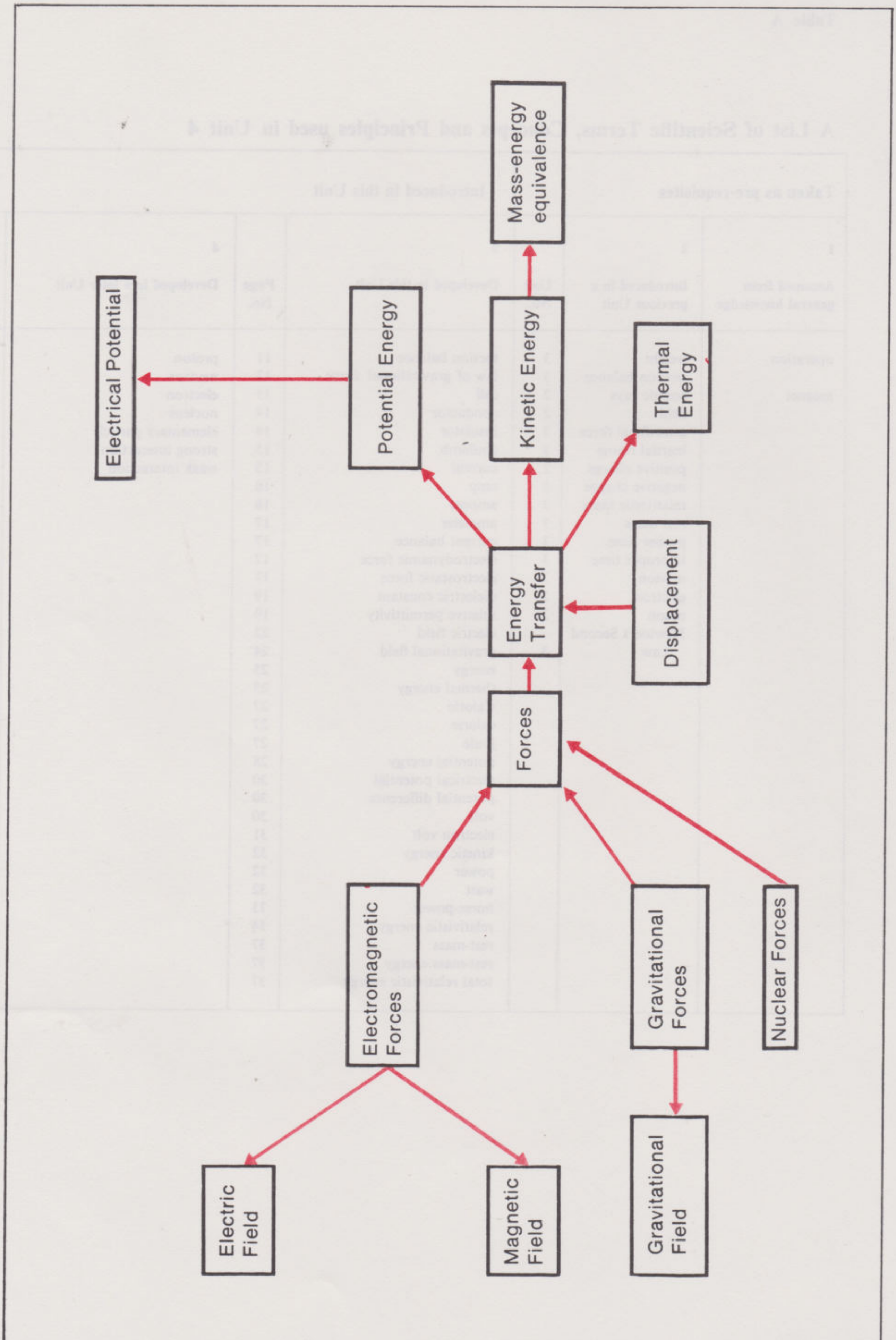
Contents

Table A

A List of Scientific Terms, Concepts and Principles used in Unit 4

Taken as pre-requisites			Introduced in this Unit			
1	2		3		4	
Assumed from general knowledge	Introduced in a previous Unit	Unit No.	Developed in this Unit	Page No.	Developed in a later Unit	Unit No.
operation	weight	3	torsion balance	11	proton	6
	newton balance	3	law of gravitational force	12	neutron	6
magnet	cosmic rays	2	cell	13	electron	6
	field	2	conductor	14	nucleus	6
	centrifugal force	3	insulator	14	elementary particle	32
	inertial frame	3	coulomb	15	strong interaction	31
	positive charge	2	current	15	weak interaction	32
	negative charge	2	amp	16		
	relativistic mass	3	ampere	16		
	rest mass	3	ammeter	17		
	proper time	3	current balance	17		
	improper time	3	electrodynamic force	17		
	newton	3	electrostatic force	17		
	electron	2	dielectric constant	19		
	muon	2	relative permittivity	19		
	Newton's Second Law	3	electric field	22		
			gravitational field	24		
			energy	25		
			thermal energy	25		
			Calorie	27		
			calorie	27		
			joule	27		
			potential energy	28		
			electrical potential	30		
			potential difference	30		
			volt	30		
			electron volt	31		
			kinetic energy	32		
			power	32		
			watt	32		
			horse-power	33		
			relativistic energy	34		
			rest-mass	37		
			rest-mass energy	37		
			total relativistic energy	37		


Mass-energy equivalence
Electrical Potential
Potential Energy
Kinetic Energy
Thermal Energy
Energy Transfer
Displacement
Forces
Electromagnetic Forces
Gravitational Forces
Nuclear Forces
Electric Field
Magnetic Field
Gravitational Field

Objectives

After you have completed your study of this Unit, you should be able to:

1 Define correctly, or recognize the best definition of all the terms, concepts and principles, in column 3 of Table A.

2 (Tested in *SAQ* 1)
State how the force of gravitational attraction between two spheres depends on the mass of the spheres and on their separation and be able to make simple calculations of the magnitude of these forces.

3 (Tested in *SAQ* 2)
State the principal factors which affect the force between two current-carrying wires.

4 (Tested in *SAQ* 3)
State how the electrostatic force between two charged spheres depends on the magnitude of the charges and on the separation of the spheres and be able to perform simple calculations of the magnitude of these forces.

5 (Tested in *SAQ* 4, 5)
Perform simple calculations which convert electrical currents measured in amps and electrical charges measured in coulombs into the equivalent numbers of electronic charges.

6 (Tested in *SAQ* 6, 7)
Perform simple calculations of the magnitudes of electric fields.

7 (Tested in *SAQ* 8)
State how the energy transferred in mechanical processes is related to the force, to the distance through which the force acts, and to the angle which the force makes with displacement, and to be able to perform simple calculations of such energy transfers.

8 (Tested in *SAQ* 9, 10, 12)
Perform simple calculations of kinetic and potential energies including calculations of electrical potential energies.

9 (Tested in *SAQ* 11)
State how the power of an electrical device is related to the potential difference across the device and the current flowing through it.

4.1 Introduction

In the previous Unit you learnt that, in an inertial frame of reference, an object left alone remains at rest (Newton's First Law). You then learnt of the effect of applying forces of various magnitude to bodies of different masses; a study culminating in the formulation of Newton's Second Law. Throughout these experimental studies, the forces came either from human pushes and pulls or from wire springs. Such forces as these are hardly in any sense 'basic'; they are undoubtedly the macroscopic expression of other, more fundamental forces. You might think that to explain all the different forces with which you are familiar—ranging from muscular pushes, through the workings of a petrol engine, to what happens in nuclear explosions—it would be necessary to postulate many such fundamental forces. Surprisingly, it turns out that the entire spectrum of macroscopic forces can be explained in terms of only four basic forces: gravitational, electromagnetic and the strong and weak nuclear interactions. In this Unit we shall begin by outlining some of the main characteristics of the first three of these four forces (later Units will take up the story of how these forces explain at least some of the everyday pushes and pulls). This will lead us to look more closely at what happens when forces act and objects are made to move. We shall begin to formulate a concept of energy.

4.2 The Fundamental Forces

4.2.1 Gravitational forces

If there is one force of which we are constantly being reminded, it is the pull of the Earth on ourselves, our weight. Expressed more objectively, what we are saying is that there is an attractive force between a large lump of matter (the Earth), and a smaller lump (ourselves). But do *any* two lumps of matter attract each other?

We all know that gravitational attraction does not, for example, cause the potatoes in a polythene pack to adhere together and that even massive objects like buildings or mountains exert no detectable physical pulls on ourselves. Were such pulls of the magnitude of a newton we would, of course, be very conscious of their presence. However, before concluding that, aside from our weight, gravitational forces are undetectable in everyday situations, it might be wise to perform a few experiments where the forces are very consciously sought.

The following experiments may sound like poor party games, but they are perfectly respectable scientific investigations. However, if you are 'certain' you know the conclusions in advance, there may be little point in performing these particular experiments; instead encourage some totally unsuspecting person to carry them out.

Experiment 1
The aim of this experiment is to detect gravitational forces. Hold something massive in each hand. Bring the objects closer together and try to decide if they attract each other. You should use a variety of objects, such as, for example, books, rocks, bags of potatoes, even cabbages; the effect may well depend on what the objects are made of. After all, our chemical composition and that of the Earth are very different. Another worthwhile variant is to close your eyes and ask someone else to bring these objects up to your outstretched hand. You, by sensing any pull on your hand, may be able to tell if the object is there.

So, although the Earth pulls on us, there is no gravitational attraction between two much less massive pieces of matter? No, it could simply be that the forces between our objects are less than the forces we can detect.

Experiment 2
What is the smallest force your hands can detect? One way to find out is to get an average apple which will weigh about 1 N and slice it up into progressively smaller pieces (counting the divisions as you go) until, with your eyes closed, you just cannot tell if a piece is on the palm of your hands. (Try shaking your palm around.) If you can sense, for example, the weight of a one hundredth part of an apple then you will be sensing a force of 10^{-2} N. Another technique is to use a pad of paper whose weight you guess, to an order of magnitude, prior to tearing it up into successively smaller pieces. Of course the experiment is a crude one, but we are only after an order of magnitude value of the minimum force you can respond to.

You may well be able to detect forces of 10^{-4} N to 10^{-6} N, and yet have failed to detect any evidence for gravitational attraction between your experimental objects. A more sensitive force-measuring device than ourselves is clearly required.

One such device, known as a *torsion balance*, is shown in Figure 1. Basically it consists of a fine quartz wire, W, carrying a light horizontal beam, B. A small force applied at either end of the beam will cause the wire to twist in much the same way as a gentle breeze can blow a swing door open. Some such balances can be made to respond to forces as small as 10^{-12} N. As you can see in Figure 1, two lumps of matter, of equal mass m_1, are hung from the ends of the beam; if they move, the beam will move. Two fixed lumps of matter, of equal masses, m_2, are then placed on opposite sides of the beam, each at a distance r from the moveable masses. Any attractive forces between the fixed and the moveable masses will cause the beam to rotate and the wire to twist. And the wire does twist; so gravitational forces do exist, aside from the Earth.

torsion balance

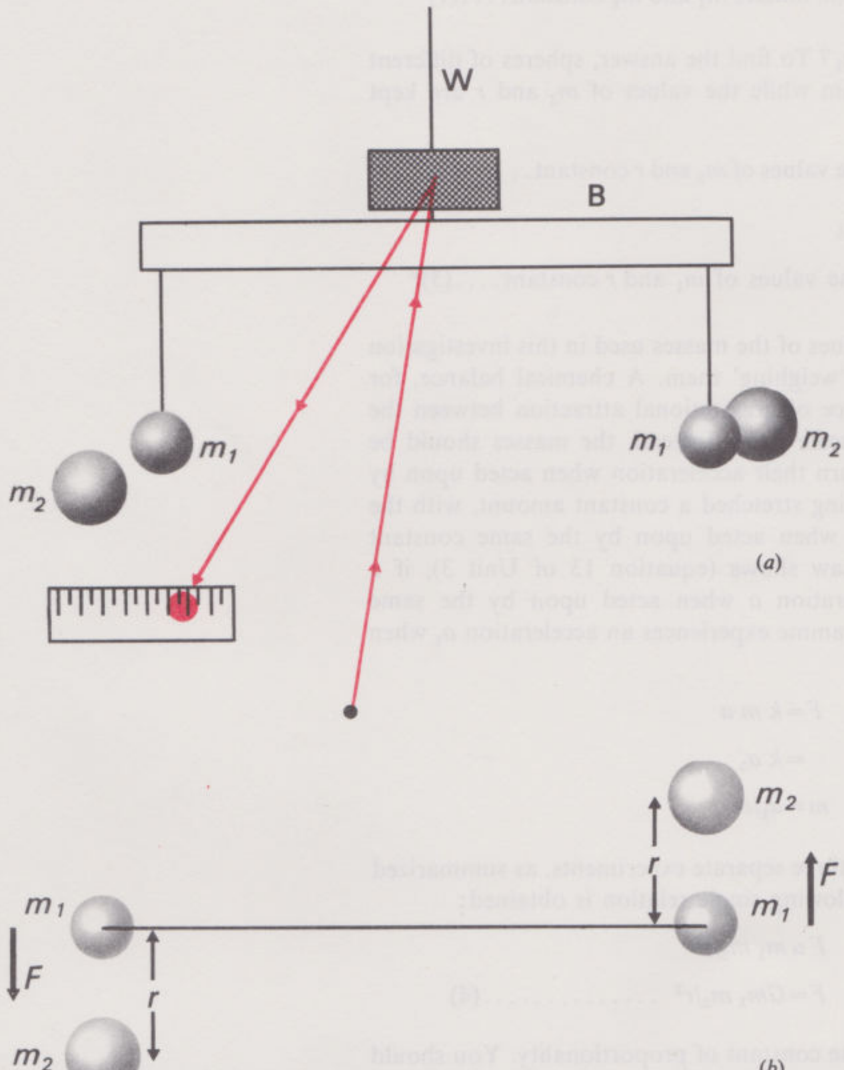


Figure 1 Gravitational forces studied by means of a torsion balance. (a) A light horizontal beam, B, carrying the masses m_1*, is hung from a fine quartz wire W. A light beam reflected off the mirror onto a scale enables the rotation of the beam to be measured. (b) A plan of the apparatus.*

The obvious next question is how does the force of attraction depend on the separation of the masses. But, should the objects happen to be, say, a bag of potatoes and a bunch of bananas, how will this separation be measured?

What shape objects should we use to make it easy to specify how far they are apart?

Uniform spheres are symmetrical and have well-defined centres and so are preferable to objects like bunches of bananas. Therefore spheres will be employed, which should be preferably of a high density material like lead or gold so that there is plenty of matter around. There is plenty of matter in the Earth and it pulls hard on us.

To study how the force of attraction F between the fixed and the moveable spheres of mass m_1 and m_2 is related to their separation r, one need simply study how the angle of twist of the wire (normally measured by means of a light beam reflected off a mirror that is mounted on the wire) varies with the positioning of the fixed spheres. The result is that, when r is doubled, F, as deduced from the calibrated twist, goes down by a factor of four. When r is trebled, F is reduced ninefold. In general,

$$F \propto 1/r^2 \text{ keeping the masses } m_1 \text{ and } m_2 \text{ constant} \ldots\ldots (1)$$

How does F depend on, say, m_1? To find the answer, spheres of different masses are hung from the beam while the values of m_2 and r are kept constant. The result:

$$F \propto m_1 \text{ keeping the values of } m_2 \text{ and } r \text{ constant} \ldots\ldots (2)$$

When m_2 is varied the result is:

$$F \propto m_2 \text{ keeping the values of } m_1 \text{ and } r \text{ constant} \ldots\ldots (3)^*$$

As a matter of principle the values of the masses used in this investigation should not be determined by 'weighing' them. A chemical balance, for example, makes use of the force of gravitational attraction between the Earth and the objects in the scale pans. Instead, the masses should be determined by comparing in turn their acceleration when acted upon by a constant force, such as a spring stretched a constant amount, with the acceleration of a kilogramme when acted upon by the same constant force. As Newton's Second Law shows (equation 13 of Unit 3), if a mass m experiences an acceleration a when acted upon by the same constant force F and the kilogramme experiences an acceleration a_1 when acted upon by F then

$$F = k\,m\,a$$

$$= k\,a_1$$

or

$$m = a_1/a.$$

Gathering up the results of the three separate experiments, as summarized in equations 1, 2 and 3, the following single relation is obtained:

$$F \propto m_1\, m_2/r^2$$

or

$$F = Gm_1\, m_2/r^2 \quad \ldots\ldots\ldots\ldots\ldots (4)$$

where G has been written for the constant of proportionality. You should satisfy yourself that equation 4 is indeed a valid synthesis of equations 1, 2 and 3. (Keeping m_1 and m_2 constant, for example, reproduces equation 1.) Equation 4 cannot, of course, be expected to describe how, say, two cylinders interact. Indeed the $1/r^2$ dependence (equation 1), the so-called inverse square law relation, would not have been found had cylinders or any other shaped objects been employed. The simplicity of the end result is, in itself, a valid enough reason for employing spheres.

* *You might well have been able to predict that equation 3 should have the same form as equation 2. Were the two equations different, the measured twist would depend on which mass was hung from the beam and which was fixed—Nature would be 'lopsided'. As you will learn from later Units, much modern research in nuclear physics is in fact devoted to searching for evidence of 'lopsidedness' in Nature, or its absence.*

As the experiments have been carried out with known masses at known separations, they produce the value of G directly. For example, when a lead sphere of mass 7.4 kg is placed 7.0×10^{-2} m from a gold one of mass 2.6×10^{-3} kg the force of attraction is 2.62×10^{-10} N.

Using this data, deduce the value of G for yourself.

Rearranging equation 4 gives

$$G = Fr^2/m_1 m_2$$

$$= (2.62 \times 10^{-10}) \times (7.0 \times 10^{-2})^2 / 7.4 \times 2.6 \times 10^{-3} \quad \text{N m}^2 \text{ kg}^{-2}$$

$$= 2.62 \times 49.0 \times 10^{-14} / 7.4 \times 2.6 \times 10^{-3} \quad \text{N m}^2 \text{ kg}^{-2}$$

So $\quad G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.

Since 1 N = 1 kg m s^{-2} (Unit 3, p. 30), this result may be written as $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}. (If these manipulations of units worry you read section 3 of the handbook *The Handling of Experimental Data*.) What is interesting is not the particular numerical value of G but the fact that, within the limits of experimental error, G is constant for a wide range of materials. This constancy suggests that we are indeed dealing with something basic.

***SAQ* 1**
Make an order-of-magnitude estimate of the force of gravitational attraction between two adult human beings when standing side by side. Using a relation derived for spheres is bound to make the answer suspect; guesses of human masses correct to a factor of two or so will therefore be quite adequate. The problem is worked out on p. 45.

The force is about 10^{-5} N.

Compare this force with the minimum force that *you* could detect in experiment 2.

Off-hand it looks as if we might *just* be able to detect the presence of objects of about human mass when we are close to them—or to detect larger objects which are further away. It is not hard to prove that similar magnitude pulls can be expected in mountainous country! Indeed an early method of determining G was to measure the angle which a plumb line (carrying a spherical bob) made with respect to the vertical when it was located close to mountains. In a later Unit (22) you will learn how a detailed study of the Earth's gravitational pull can provide important clues about the Earth's structure.

4.2.2 Electromagnetic forces

Early in the nineteenth century, it was discovered that, if one dipped metal plates into certain solutions, interesting things happened when the plates of these *cells*, or batteries, were connected together in various ways. The sort of circuit which was employed is shown in Figure 2 (a) where the batteries are indicated by pairs of lines of unequal length; unequal to

cell (battery)

emphasize that the two plates have different compositions. The circuits are completed by lengths of metallic wire which run reasonably close to each other. Switches are incorporated as a convenient way of making and breaking the circuit. We may summarize the experiments (some of which you will see being performed in the TV programme of this Unit) and their findings, as follows:

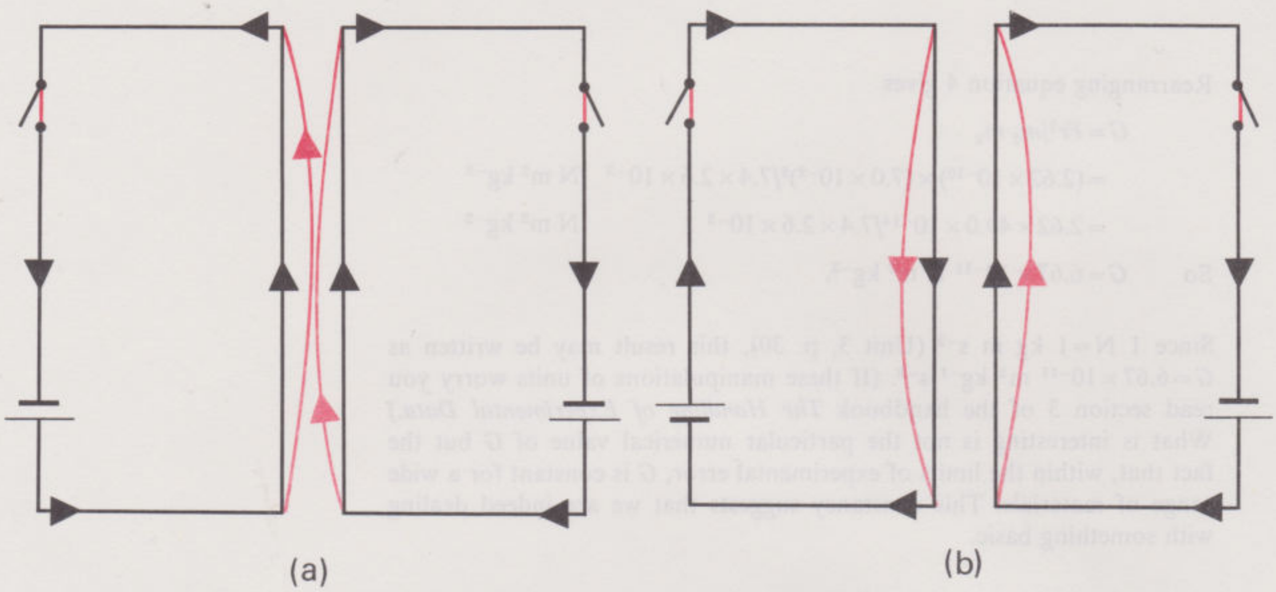


Figure 2 Electrodynamic forces. Depending on the sense in which the batteries are connected up, the force between the wires may be attractive, as in (a), or repulsive, as in (b).

1 When both switches are closed, each wire is attracted towards the other. The force is only present when *both* switches are closed; open one and the wires will cease to attract. Since the masses of the wires do not change when the switches are closed, the force can hardly be gravitational in origin. So some other force must be responsible.

2 When the connections to one of the batteries are reversed as in Figure 2 (b), and the first experiment is repeated, the wires, instead of being attracted together, are now pushed apart. Of course if the connections to the second battery are also reversed, the circuit will be back to that of Figure 2 (a) (turned upside down), so this further experiment need not be performed. (Strictly speaking, however, this is just the sort of experiment that *should* be performed, *a priori*, there is no reason why the experiment might not behave differently when 'upside down'. In this case, however, it behaves the same.)

3 When the number of batteries in the circuits is increased, the forces of attraction or repulsion are also increased.

4 When the wires are moved further apart the forces decrease. Changing the angle between the wires also changes the forces.

5 When the experiments are performed with the circuits connected up by lengths of plastic, nothing moves, i.e. no forces are present. Materials like plastics which give no effect are called *insulators*; those that do, *conductors*.

insulators and conductors

6 When the medium in which the wires are located is changed the forces may change. For example, lumps of iron placed between the wires will increase the forces.

Apart from heating up, the wires do not change in any obvious way when the switches are closed. However, there is a simple demonstration which does suggest that the internal state of the wires does change. As you will see in the TV programme, the evidence suggests that something flows through conductors that are connected across electrical supplies, such as batteries. One can therefore, as in Figure 2, put in arrows indicating the direction of flow of this 'something'*.

The Broadcast Notes contain further details of the experiment. Unfortunately this experiment gives no definitive answer as to the nature of this 'something'. It does not tell us, for example, whether it is something discrete (like a stream of particles) or something continuous (like a so-called 'classical' fluid: one with no obvious atomic character). Whatever the nature of this 'something', it would be very convenient to be able to specify the magnitude of the *current*, in other words how much of this 'something' is flowing past a fixed point in the wire per second. Indeed, the conventional way of speaking of an electrical current as so many *coulombs of charge* flowing past a point per second is very convenient, since we could, if we wished, imagine 'coulombs of charge' to stand for either 'number of particles' or 'cubic metres of fluid', depending on how we picture the 'something'.

current

coulomb

To frame an operational definition of unit current one must, of course, select some property of a current-carrying wire and then decide what to call the current when this property has such and such a value. We might decide that if the paddle-wheel you saw in the TV demonstration takes a defined number of seconds to move a defined distance, then one coulomb of charge passes through the tube every second. However, it is difficult to obtain reproducible results with such a tube. The internationally agreed definition of the unit of current in fact makes use of the forces between current-carrying wires. In effect, it prescribes what we are to take as the magnitude of the currents once we have measured the force between two short lengths of wire at a fixed position relative to each other. Force measurements are easily made with a newton balance (which has, in its calibration, involved the basic units of mass, length and time). Furthermore, such balances give closely reproducible results. Here is an example of how the international definition works in practice with one particular experimental layout.

We wish to specify the current, i, flowing in a wire in terms of the force between this wire and an adjacent current-carrying wire. Perhaps the simplest arrangement is to bend a single wire back on itself so that there are two parallel wires, each carrying the current i. This layout is shown in Figure 3 (p. 16), where the wires run on sliders to prevent the bending of the wires which occurred with the circuit of Figure 2. Newton balances connected to the wires enable the force of attraction between them to be measured accurately. *With this particular geometrical arrangement* it has been agreed that if there is a force of 2×10^{-7} N per metre of length between two infinitely long straight parallel conductors of negligible cross-section placed 1 m apart in vacuum, then there is a *defined* current of 1 coulomb of charge (written 1 C) passing a point in each wire per second. If the force is 8×10^{-7} N per metre the current is 2 C per second.

If it is 18×10^{-7} N per metre the current is 3 C per second. As these examples show, the force is taken to be proportional to the product of the currents in each wire; in this particular layout the currents will necessarily be equal.

* *Because of the conventions of the subject, the directions indicated by the arrows are in fact in the opposite sense to the direction of movement of the paddle-wheel seen in the TV demonstration. Before experiments like the paddle-wheel were performed, one could only guess the direction of the current. In fact, the wrong sense was originally chosen, but this wrong choice is still retained in indicating the current direction.*

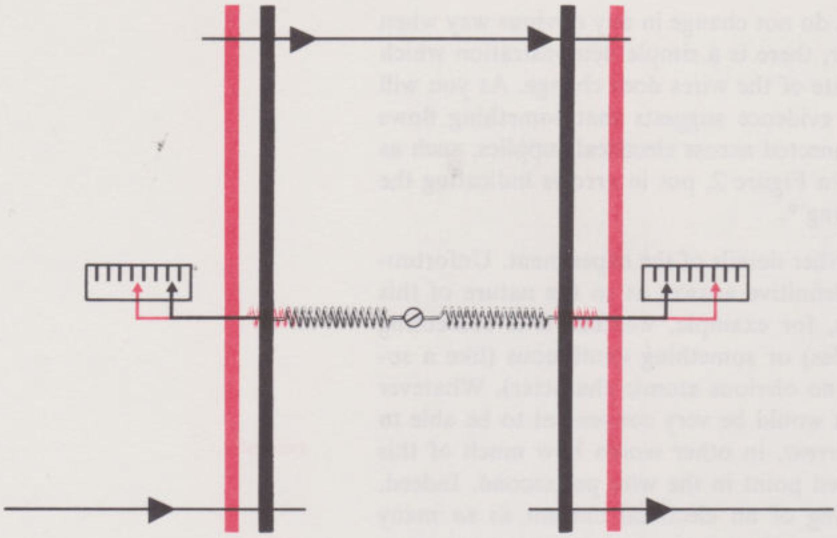


Figure 3 A schematic current-balance. The force of attraction between the two wires, each carrying the same current, i, *is measured by a newton balance.*

You should not attempt to memorize the particular magnitude of the force that must exist between the two wires for the current to have a defined value of 1 coulomb per second, any more than you should try to memorize the number of wavelengths involved in the definition of the metre. You should, however, appreciate that the definition of the coulomb is every bit as fundamental as those of the metre, the kilogramme, and the second. All physically measurable quantities can be expressed ultimately in terms of suitable multiples of the basic units of mass, length, time and charge.

Equal currents flow in each of two long parallel conductors placed 1 m apart in vacuum. If the force between the wires is 160×10^{-7} N when measured over a 5 m length what current flows in each wire?

The force per unit length of wire is $160 \times 10^{-7}/5 = 32 \times 10^{-7}$ N, which is 16 times the force produced by unit current. Therefore, as the force is proportional to the square of the current in one of the wires (which is the same thing as the product of the currents in each wire), $\sqrt{16} \times 1\ \mathrm{C} = 4\ \mathrm{C}$ flows past a point in each wire per second. As it is rather tedious to have to keep repeating 'a coulomb of charge passing a point in the wire per second', this is conventionally shortened to *ampere*, or *amp* (written A). So in this example the current is 4 A. While it is possible to measure such small forces as 10^{-7} N, sensitive balances are required. Practical current meters measure not the force between two single lengths of parallel wires but the force between two coils of wire (each consisting of many such parallel wires all carrying the same current). Figure 4 shows such a meter or 'current balance', where the interacting coils are placed one inside the other (one of the coils is removed in Figure 4). As the coils are connected up one after the other, in *series*, the same current flows through each one. The direction of the current is such that the pair of coils at one end of the balance attract, while the pair at the other end repel, with the result that the beam carrying the coils rotates. To bring the beam back to the horizontal, weights are hung at appropriate places on the beam. From the position of these weights, the force acting between the coils can be calculated and hence the current can be deduced.*

ampere

** The international definition relating current to force does not attempt to cover every possible type of wire layout. Instead, it deals with two short lengths of current-carrying wire. To discuss any practical layout one must 'integrate' the relation dealing with the short lengths of wire. We have quoted the integrated relation in the case of the two long parallel wires.*

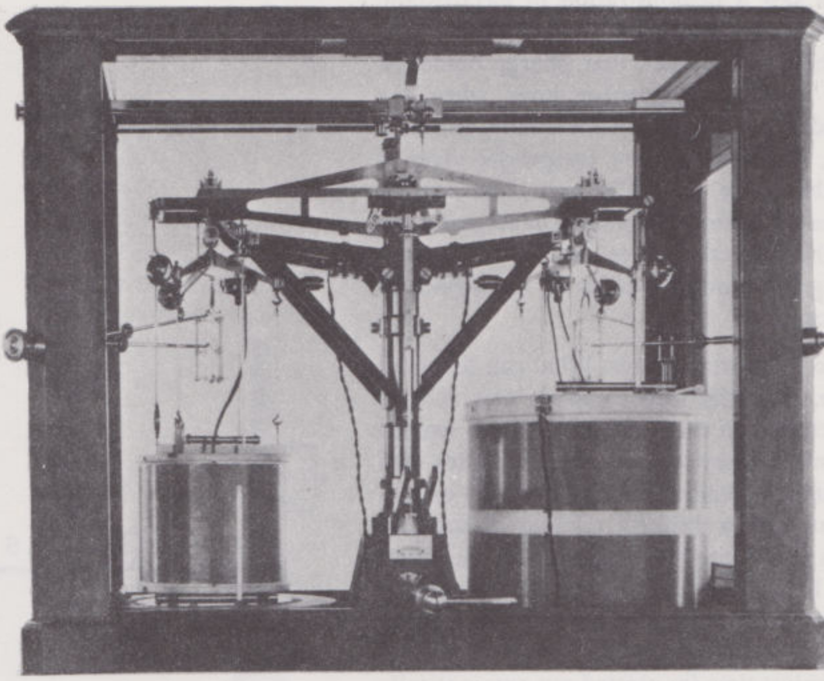


Figure 4 *A practical current-balance. A coil is hung from each arm of the balance and the forces between these coils and fixed outer coils (one of which is removed) are attractive at one end of the balance and repulsive at the other. Weights are added to the beam of the balance so as to return it to the horizontal position.*

current balance

ammeter

The type of 'ammeter' most commonly found in science laboratories measures not the force between two current-carrying coils but the force between one such coil and a magnet. As will be shown in Unit 23 a magnet, in fact, behaves like a current-carrying coil. This type of meter must, however, be calibrated against a current balance. Figure 5 shows a close-up of such a meter.

electrodynamic force

electrostatic force

In all these experiments involving current-carrying wires, the electric charge has been moving relative to the person carrying out the experiment or, put more formally, relative to the force-measuring device. Such forces may, for obvious reasons, be labelled as *electrodynamic*, although, for historical reasons, they are more usually called *magnetic* forces. However, when charges are at rest relative to the experimenter, an apparently different force appears; the so-called *electrostatic* force. The interrelation of these two forces is discussed further in the radio programme of this Unit.

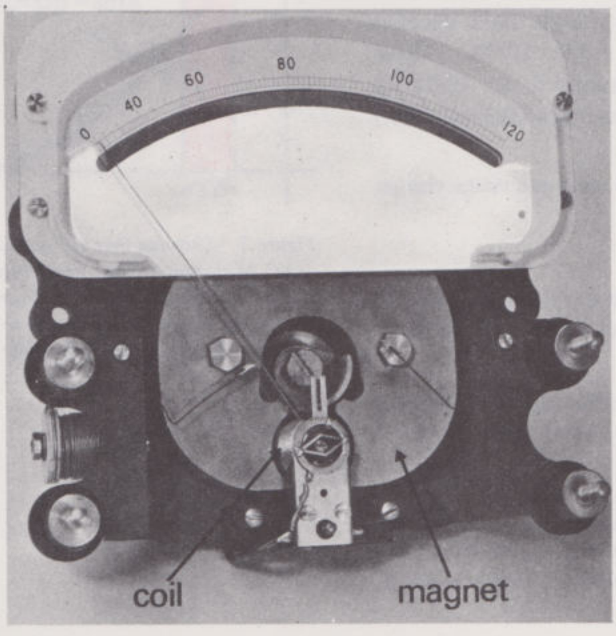


Figure 5 *A practical ammeter. The force between the current-carrying coil and the magnet causes the coil to rotate about the axis. A pointer is attached to the coil. This particular instrument is calibrated in milliamps* (mA).

In the television programme you saw a demonstration of electrostatic forces. Using the circuit of Figure 6, the experiment showed that charge could be collected on metal spheres, or rather that charge could be transferred from one sphere to another. Remember how, when the switch S was closed, a current did appear briefly in the circuit and as a result of this transfer of charge, the upper sphere (suspended from a spring) was attracted towards the lower one. Moreover, *the spheres continued to be attracted even when the current had ceased flowing* and the switch had been reopened, so the attractive force cannot be electrodynamic in origin. Nor can it be gravitational in origin as the masses of the spheres have not changed perceptibly.

We have observed a new force, the electrostatic force. What can be found out about it? How, for instance, does the force between two spheres depend on the magnitude of the charge on each sphere and their separation? To make the investigation, we will have to use a set of spheres with different charges. These can be simply obtained by repeating the experiment you saw on TV (Fig. 6), but with a different number of batteries in the circuit. With more batteries in the circuit the area under the graph of current against time is greater. In any particular experiment the charge which is transferred from one sphere to another is represented by *the area under the graph* of current against time. To see why this is so we approximate the smooth plot of current against time (as plotted out automatically on the cathode-ray oscilliscope) by a succession of steps (Fig. 7). We suppose that during an interval δt the current is constant with a value I, that is, there is I coulomb of charge per second flowing from one sphere to the other during this interval. If we then multiply the current I (in C s^{-1}) by the time, δt (in s) for which it flows, we get the total charge δQ (in C) flowing in this interval. (E.g. if $\delta t = 10^{-6}$ s and $I = 5 \times 10^{-3}$ C s^{-1}, then $\delta Q = I \times \delta t = 5 \times 10^{-3}\ \mathrm{C\ s^{-1}} \times 10^{-6}\ \mathrm{s} = 5 \times 10^{-9}$ C.)

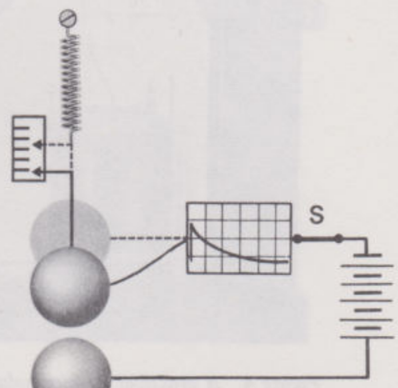


Figure 6 Electrostatic forces. On closing the switch S the two metallic spheres attract each other. The cathode-ray oscilloscope enables the charge transferred from one sphere to the other to be measured.

But the product of I and δt is nothing more than the area under this particular step (the height times the base). The same argument holds for each and every step, so the total charge transferred in the experiment from one sphere to the other is the total area under the curve, which can be measured (e.g. by the 'counting squares' technique).

By repeating the charging experiment with different numbers of batteries in the circuit, we can acquire a large selection of spheres of known charge. In all experiments of this kind, one of the spheres is always found to have lost charge, the other to have gained it. Those spheres which have gained charge are conventionally labelled positive (+), those which have lost charge are labelled as negative (−).*

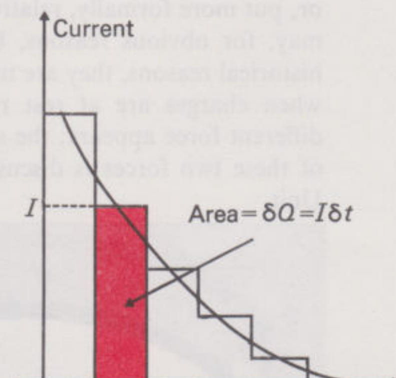


Figure 7 Showing that the area under a graph of current against time equals the charge δQ transferred. In an interval δt the charge transferred is Iδt where I is the current; Iδt is the magnitude of the shaded area.

A collection of charged spheres obtained in this way might have charges of say $+1.5 \times 10^{-8}$ C, -1.5×10^{-8} C, $+3.0 \times 10^{-8}$ C, -3.0×10^{-8} C. (These are the sort of charges you collect with a few hundred car batteries connected to spheres with the diameter of about 0.1 m placed 0.3 m apart.)

In Unit 2 it was stated that like charges repel each other and unlike charges attract. How might we verify this?

We could hang a sphere of one sign charge at the end of a newton balance and bring up another of similar or opposite sign and watch the result.

How should we now proceed with a systematic study of how the force F between two spheres depends on their charges, Q_1 and Q_2, and on the separation r between their centres?

* *As already mentioned (p. 15), the actual charge carriers really move in the opposite sense to that suggested by the conventions. These carriers (electrons) possess a charge which is taken to be negative. A gain of positive charge is therefore, strictly speaking, better described as a loss of negative charge.*

If you recall the techniques used in studying gravitational forces (section 4.2.1) you may have the answer. In exact analogy with our studies of gravitational forces, one finds out how F depends in turn on Q_2 (while the values of Q_1 and r are kept constant) and finally on r (while the values of Q_1 and Q_2 are kept fixed). The combined results of these experiments show that . . .

$$F \propto Q_1\, Q_2/r^2$$

or

$$F = \left(\frac{1}{4\pi\epsilon_0}\right) Q_1\, Q_2/r^2 \ldots\ldots\ldots (5)$$

where the constant of proportionality has been written as $1/4\pi\epsilon_0$ (ϵ is pronounced 'epsilon'). The constant is given this rather awkward form because it simplifies the notation in more advanced problems. So long as ϵ_0 is a constant, $1/4\pi\epsilon_0$ will, of course, be constant.

Since the experiments have employed known charges at known separations and the resulting forces have been measured, this data is sufficient to determine $1/4\pi\epsilon_0$ just as the gravitational data determined G.

With two spheres of charge $Q_1 = Q_2 = 1.4 \times 10^{-8}$ C (deduced from the area under the graph of current against time) $F = 0.217 \times 10^{-5}$ N when $r = 0.90$ m. Deduce $1/4\pi\epsilon_0$.

From equation 5

$$\frac{1}{4\pi\epsilon_0} = \frac{Fr^2}{Q_1\, Q_2}$$

$$= \frac{0.217 \times 10^{-5} \times (0.9)^2}{1.4 \times 10^{-8} \times 1.4 \times 10^{-8}}\ \frac{\text{N m}^2}{\text{C C}}$$

$$\therefore \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N m}^2\text{ C}^{-2}$$

Since N means kg m s^{-2} we can, if we wish, write

$$\frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ kg m}^3\text{ s}^{-2}\text{ C}^{-2}$$

Unlike G, the value of which is apparently uninfluenced by the medium in which the experiment is carried out, the electrostatic constant of proportionality varies from medium to medium. For example, if the surrounding of two charged but otherwise isolated spheres is changed from air to glycerine, the force between them is reduced by a factor of 42.5.*

4.2.3 Nuclear forces

In later Units we shall be presenting evidence that all matter is made up of atoms, and that these atoms have their own complex structure, consisting of a central *nucleus*, with a diameter of some 10^{-15} m, surrounded by an electron cloud which extends out to a diameter of around 10^{-10} m. We shall be presenting evidence that the nucleus consists of roughly equal numbers of protons and neutrons. Protons are particles with a charge of the same magnitude as that of the electron but of opposite sign and with a mass of 1.67×10^{-27} kg, which is nearly 2 000 times the mass of the electron. Neutrons are uncharged particles with approximately the same mass as protons. How, we may ask, can such a nucleus remain stuck so firmly together? Surely the protons will be driven apart out of the nucleus by electrostatic repulsion? Not necessarily. Perhaps the force of gravitational attraction, F_g say, between the two protons is at least as great as the force of electrostatic repulsion, F_e say. We can check this by seeing if F_g/F_e has a value of unity or greater.

atom

nucleus

proton

neutron

* *In terms of equation 5 this means that the constant of proportionality is reduced by a factor of* 42.5. *To cover such general cases equation 5 is normally written as*

$$F = \left(\frac{1}{4\pi\epsilon_r\epsilon_0}\right) Q_1\, Q_2/r^2$$

The constant ϵ_r is known as the dielectric constant *or* relative permittivity *of the medium. The value of ϵ_r for glycerine is therefore* 42.5. *Of course, in a vacuum ϵ_r has a value of* 1 *exactly. In air, ϵ_r =* 1.0005.

relative permittivity

Treating the protons as spherical particles of mass *m* and charge *e*, write down expressions for F_g and F_e. Next evaluate the ratio F_g/F_e.

From equations 4 and 5:

$F_g = Gm^2/d^2$, where d is the separation between two protons.

and $$F_e = \left(\frac{1}{4\pi\epsilon_0}\right) e^2/d^2$$

So $$\frac{F_g}{F_e} = \frac{Gm^2}{d^2} \bigg/ \frac{e^2}{4\pi\epsilon_0 d^2}$$

$$= \frac{Gm^2}{d^2} \times \frac{4\pi\epsilon_0 d^2}{e^2}$$

$$= 4\pi\epsilon_0 Gm^2/e^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Notice how the d^2 cancels out; a direct consequence of the fact that the gravitational and electrostatic forces have the same spatial dependence.

So this argument does not depend on the separation between nuclear protons.

Substituting into equation 6 the values:

$$\frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2},$$

$$e = 1.60 \times 10^{-19} \text{ C},$$

$$m = 1.67 \times 10^{-27} \text{ kg},$$

gives: $$\frac{F_g}{F_e} = \frac{6.67 \times (1.67 \times 10^{-27})^2}{8.99 \times 10^9 \times 10^{11} \times (1.60 \times 10^{-19})^2} \quad \frac{\text{N m}^2 \text{ kg}^{-2} \text{ kg}^2}{\text{N m}^2 \text{ C}^{-2} \text{ C}^2}$$

i.e. $$\frac{F_g}{F_e} \approx \frac{1}{10^{36}}$$

The repulsive force is some million million million million million million times larger than the attractive force. Without any doubt some new basic force operates inside the nucleus! Clearly it must be an attractive force. It must also be short-ranged, i.e. it must not extend far outside the nucleus, for if it were to extend further we would be unaware of the much weaker gravitational and electromagnetic forces in the world around us. (In fact, within the short range of the nucleus, this force is some hundred times stronger than the electromagnetic force.) Atomic nuclei have neutrons as well as protons in them and are *stable*; that is, they do not in general break up spontaneously, so the nuclear force must act on the neutrons as well as the protons. Whether the nuclear force acts just as strongly on neutrons as it does on protons remains to be seen (in Unit 31).

The force, some of whose properties we have just listed, is usually referred to as the nuclear *strong interaction*. To account for certain other aspects of the behaviour of sub-nuclear *elementary particles* such as, for example, the process leading to the decay of the muon, it is necessary to postulate yet another basic force, the *weak interaction*. Discussion of this is deferred until Unit 32.

strong interaction

weak interaction

4.3 Fields

4.3.1 The field concept

You will recall how, in Unit 2, we introduced the concept of a *field* as a means of accounting for action at a distance. An analogy was drawn with the behaviour of a stretched rubber membrane; distort it in one place and it distorts all around. But a field, remember, is only a model, introduced to explain why, for example, one charge should attract or repel another one placed some distance away.

Here is yet another model of a field which presents an alternative viewpoint and serves to recall the salient points of the earlier discussion in Unit 2.

If you were to take two corks and place them on a pool of water and were then to wiggle one cork up and down, it would not be long before the second cork also wiggled up and down.

Why?

It is a trivial question, because in our imaginations we can see that what happens is that one cork disturbs the water and the second one experiences the disturbance. But now suppose that bad eyesight prevents you seeing the water.

How would you describe the experiment?

You would see one cork being moved up and down and a second one going up and down in sympathy. You would conclude that there is a direct interaction between the corks; action at a distance.

Hold this text at arm's length. Certainly you can feel the Earth pulling on it. Perhaps you conclude that there is a direct interaction between the Earth and the book. In imagination, watch a couple of current-carrying wires in close proximity to each other—a direct interaction? Those charged spheres—are they really acting on each other at a distance? Might it not simply be that we are blind to these forces? Perhaps what really happens is that the Earth creates some disturbance which then interacts with the book. Perhaps one current-carrying wire creates a disturbance which the other current-carrying wire senses. Perhaps one charged sphere creates a disturbance which the second one merely responds to. The agony of these speculations is that, like A. A. Milne's Pooh, we never can tell. If you have poor eyesight, the only way to find out if the water is disturbed is to place something on it and look for an interaction. The only way we can detect whether a mass, a current-carrying wire, or a charged sphere produces any disturbance is to introduce another mass, another current-carrying wire, or another charged sphere. But we could just be witnessing a direct interaction—we never can tell.

No one is going to dictate your choice of a field model. In the past, such models have included springs, bent tubes, and even gear wheels that fill space. Of course, the more you use any one model, the more deeply convinced you may become of its reality. Keep several at your finger tips and you may avoid falling into this trap. The following experiment was

originally designed to demonstrate to schoolchildren just how a particular model, that of a stretched spring, can account for the forces of gravitational attraction. It can be such a convincing demonstration that having performed the experiment one can readily refute all suggestions that gravitational attraction just might not, after all, be due to a stretched spring. As a tale with a moral, the experiment is well worth performing, in spite of it being of the nature of a children's game.

First decide to accept the model that the interaction between the Earth and say a book is attributable to an invisible spring. Find a massive book and place it on the palm of your hand, keeping your arm fully outstretched. Now close your eyes and lift the book at arm's length, saying as you do so, 'I am stretching a spring'. As you lower the book say, 'I am relaxing the spring'. Keep this up for some time and you are likely to become deeply convinced of the reality of the spring connecting the book to the Earth.

It is only an experiment in self-hypnosis, but despite this, or rather because of this, it makes a worthwhile comment on the reality of models.

4.3.2 The electric field

If we are to make full use of the concept of a field, we must put it on a more quantitative basis. Fortunately, while there are diverse qualitative pictures of what a field is 'like', there is a fair amount of agreement among scientists as to how to express field magnitudes (strengths).

It is an observed fact that a sphere possessing a charge Q will exert a force of magnitude of $Qq/4\pi\epsilon_0 r^2$ on a sphere of charge q placed a distance r away. It is known too that the force is directed along a line connecting the centres of the spheres, that it is attractive when Q and q are of opposite sign and repulsive when Q and q have the same sign. Let us imagine that what actually happens is that one charge creates a field with which the second one subsequently interacts. Figure 8 shows a schematic diagram of the field. If you like, you may think of the lines as springs waiting to push on any other charged body. You may even care to imagine that introducing this other charged body (which of course will have its own spring system) cuts the tapes which prevent the springs acting on nothing. The mechanism is your choice.

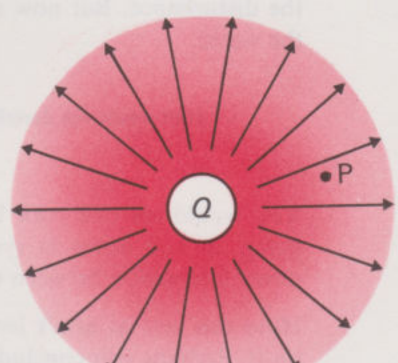


Figure 8 A representation of an electric field produced by a charge Q. The lines represent the field. The arrow indicates the direction of the field.

Mechanisms apart, we must agree on a definition for the value of the field at, say, a distance r from a charge Q. Following tradition, the electric field E at any point is defined as the *would-be force per unit positive charge placed at the point*. We have taken the somewhat unusual step of incorporating the words 'would-be' to emphasize that we are assuming the disturbance is still there even when we are not sampling it. 'Per unit positive charge' means that we calculate what the force would have been if we had placed a charge of $+1$ C at the point in question. For example, if we found a force of 10^{-2} N on a charge of $+10^{-3}$ C the force on $+1$ C would have been 10^{-2} N$/10^{-3}$ C$=10$ N C^{-1}. It is like calculating the speed in metres per second of a car that has been observed to travel 3 m in 10^{-1} s; its speed is 3 m$/10^{-1}$ s$=30$ m s^{-1}. In representing the field schematically, as in Figure 8, it is usual to draw in arrow heads to show the field direction, i.e. the direction of the force on the positive 'test charge' introduced at the point.

From our definition, it follows that the electric field, E, at a point P, at a distance r from a single charge Q, is

electric field

$$E=\frac{F}{q}$$

where F is the force that a charge q would experience when introduced at point P. Substitution for F from equation 5 gives

$$E = \frac{Qq/4\pi\epsilon_0 r^2}{q}$$

i.e.

$$E = \frac{Q}{4\pi\epsilon_0 r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

What are the units of E?

From the definition, it should be 'newton per coulomb', i.e. the force per unit charge. Remembering that $1/4\pi\epsilon_0$ has units of N m^2 C^{-2} (see p. 19) the right-hand side of equation 7 has units of (N m^2 C^{-2}) . (C) . (m^{-2}) which are indeed N C^{-1}.*

SAQ 6
Calculate the electric field at a distance of 2×10^{-10} m from a charge of 1.6×10^{-19} C.

The problem is worked out on p. 46.

You should obtain an answer of 3.6×10^{10} N C^{-1}. Perhaps you recognized the figures in the problem. What you have just calculated is the field produced by a single electronic charge at a distance away of about one atomic diameter.

SAQ 7
What will be the force on a 1.6×10^{-19} C charge when introduced into a region where the electric field is 3.6×10^{10} N C^{-1}?

The problem is worked out on p. 46–47.

It is around 5.76×10^{-9} N.

This is actually the sort of attractive force that one might expect to find between two atoms in a molecule of common salt (sodium chloride). As you will learn in Unit 9, in sodium chloride the sodium atom has transferred an electron to the chlorine atom (which therefore has a charge of magnitude 1.6×10^{-19} C.) Using relatively simple concepts, we have made a quite fundamental, if rough, calculation of the attractive forces to be found in one particular, but important, molecule.

What charge configuration produces a field of 10^3 N C^{-1}?

There is no unique answer. Remembering the form of equation 7 and that $1/4\pi\epsilon_0 = 9 \times 10^9$ N m^2 C^{-2} (near enough) this field could be produced for example by a single charge of +1 C at a distance of 3×10^3 m away. Alternatively the field could be produced by four times the charge twice as far away. Or by several charges. You never can tell. Field values are experimentally measurable quantities; we do not have to know what charge configuration is producing these fields. Conversely we cannot infer the configuration uniquely.

* *Later in the Unit, we shall be introducing the term* volt (V) *as shorthand for N m C^{-1}. Therefore the units of field, namely N C^{-1}, can also be written as V m^{-1}.*

4.3.3 The gravitational field

In exact analogy with the electrostatic case, one can define the gravitational field E_g at a point as the *would-be force per kilogramme mass placed at a point*.

From this definition write down an expression for the gravitational field E_g at a distance r from a spherical mass M.

The answer, obtained by writing down the force which would act on, say, a mass, m, at the point in question (using equation 4) and then dividing the result by m to give the force per unit mass:

$$E_g = GM/r^2 \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

gravitational field

Given that the Earth has a mass of about 6×10^{24} kg and a radius of about 6.4×10^6 m, deduce the gravitational field at the Earth's surface.

This field is 9.8 N kg^{-1}. This value, calculated by substituting $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}, $M = 6 \times 10^{24}$ kg, and $r = 6.4 \times 10^6$ m into equation 8, differs slightly from the field that would be measured by, say, hanging up a 1 kg mass at the end of a newton balance. The discrepancy between the calculated and observed field values is attributable in part to the centrifugal force acting on the test mass as a result of the rotation of the Earth. This discrepancy will be discussed further in Unit 22.

4.3.4 The magnetic field

magnetic field

In giving field representation to the interaction between two current-carrying wires, it is usual to say that one of the wires produces a magnetic field which can be represented by a series of concentric rings as shown in Figure 9. The arrows, indicating the direction of the field, are drawn in such a sense that a right-hand screw (e.g. a corkscrew) rotated in this sense would advance in the direction of the current flow. The reasons for drawing rings rather than, say, radial lines are partly historical.*

The magnitude of the magnetic field at some point, P, in Figure 9 is defined as the 'would-be force per metre of wire' carrying a current 1 A when placed parallel to wire W. In a more general situation, the field magnitude is found from the maximum force on the current-carrying element (the test wire if you like) as its orientation is altered. We will have more to say about magnetic fields in later Units. Indeed, measurement of the Earth's magnetic field provides useful clues as to the Earth's structure.

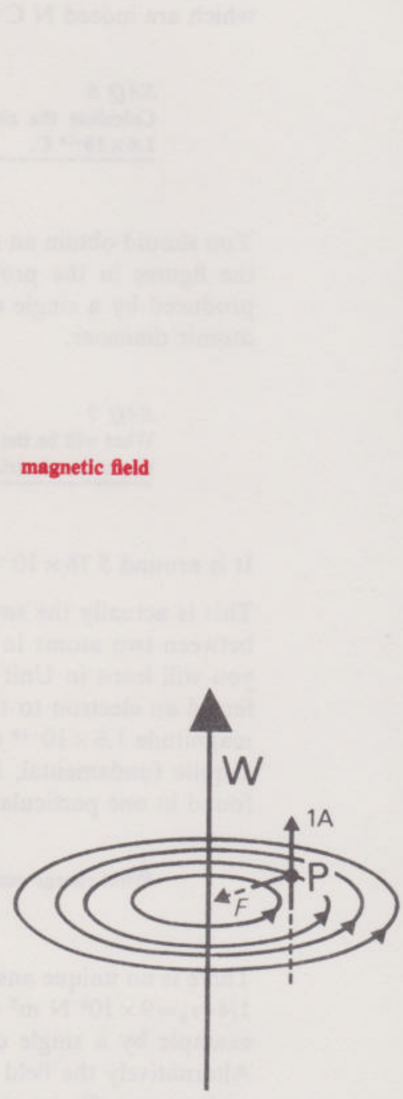


Figure 9 A representation of a magnetic field produced by a current-carrying wire, W, *showing the magnetic force* F *acting on a short length of wire carrying a current of* 1 *ampere. Note that the force is at right angles to both the current and the magnetic field, i.e. it is directed radially inwards.*

* *If a magnetic compass is moved in a circular path around such a wire, the axis of the magnet keeps tangent to the circle. This does not however prove the existence of such a field, for the magnet itself contains 'circulating' charges at the atomic level. The behaviour of the compass can be explained equally well in terms of a direct 'action at a distance' between the current in the wire and the 'currents' in the magnet.*

Section 4

4.4 Energy

4.4.1 A partial picture

In all the experiments with forces that we have described, we have taken it as axiomatic that such forces are, so to speak, available gratis. But we know as a matter of experience that we cannot, for example, keep on digging in the garden, i.e. keep on applying a force causing things to move, unless we consume fuel in the form of food. Nor is the requirement peculiar to humanity. Use a rotary cultivator if you like, but it requires petrol. An electric cultivator requires connecting to the mains. Operate such an electrical device and more fuel, perhaps coal, will be consumed in the power station. The performance of any useful everyday job of work in which a force is made to move an object (or, put more formally, in which a force moves its point of application) requires fuel in one form or another. This something, which is stored in fuels and which enables useful jobs to be done, is called *energy*. In the present context, 'useful jobs' are defined as those in which the force moves its point of application. The word fuel must not be thought of too restrictively. It might, for example, be possible to dig the garden by means of a Heath Robinson device in which a weight hung over a nearby cliff descends towards sea-level. In this case the 'fuel' is to be sought in the Earth's gravitational field; if you adopt a 'spring' model for gravitational fields the energy is stored in the stretched spring. But what happens to the energy which is contained in fuels after the fuels have been used and the job has been done? One line of investigation is to look at the 'end products'; if they can themselves do useful jobs then, by definition, at least some of the energy present in the fuel has not vanished into nothingness. To throw a puck, for example, one requires chemical energy in the form of food. Now consider the moving puck—it could be made to collide with a spring. The spring would be compressed and a compressed spring can do many useful jobs. So the moving puck had energy, *kinetic energy* as we say, energy by virtue of its movement, and this was transformed into *potential energy* in the spring, energy by virtue of the spring's configuration. But, instead of compressing a spring, the puck could have hammered a nail into a piece of wood. Some of the puck's kinetic energy would have been transformed into potential energy in the strained wood but some of it would have appeared as heat or *thermal energy*. The nail feels warmer than its surroundings and, in principle, there is no reason why this heat should not be utilized. Indeed, if you follow through any sequence of energy transfers —such as a compressed spring which works a dynamo which drives a light bulb, for example—you will always find that at each stage part of the energy is transformed into thermal form. Once the thermal energy has 'run away' into the surrounding cooler region it is no longer accessible unless we have something cooler still to transfer it to. Steam can only be used to drive a steam engine, for example, if there is a cool region for the steam to condense in. The tepid water which results might still be used to do other useful jobs, but cool water at the same temperature as the air in the room can by itself do no further useful jobs. This is not to say that the energy has disappeared; it is just inaccessible. Indeed, one naturally assumes that energy must be conserved: it must 'go somewhere'. If it does not turn up in an obvious form, one starts offering excuses in the form of new names for the 'missing' energy.

energy

kinetic and potential energy

thermal energy

conservation of energy

After a rotary cultivator has dug a garden, what has happened to the energy that was in the petrol which has been consumed?

A good deal, perhaps about half, of the energy has gone into thermal form. But the general level of a dug garden is higher than an undug one; there is gravitational potential energy in the raised earth—one could arrange for further useful jobs to be done as the earth sinks down to its former level.

4.4.2 Measuring energy transfers

Is there any way of combining the forces which appear when mechanical jobs of work are done with the distance through which these forces act, to obtain a realistic measure of the energy transfer. Should we perhaps multiply the square of the force by the cube of the distance moved, or should we . . .?

The best way to find out is by experiment.

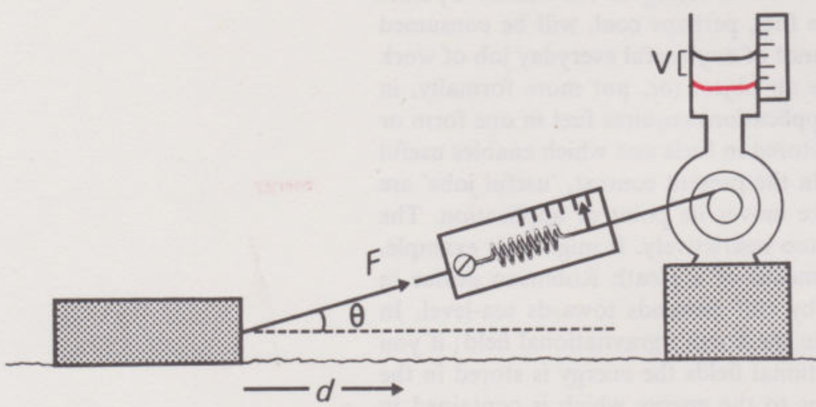


Figure 10 *Measuring energy transfers. A petrol engine fitted with a graduated tank pulls a load across rough ground. The experiment studies how the petrol consumption V depends on* F, d *and θ.*

Have a look at the apparatus shown in Figure 10. Here a petrol engine fitted with a graduated tank slowly pulls a load across rough ground, via a newton balance, transferring energy in the process *from* chemical form in the fuel *to* thermal form in the ground. We wish to know how the petrol consumption, or rather the extra volume V consumed over that used when running light, depends on the pulling force F, on the distance d through which it acts, and on the angle θ between the direction of the force and that of the movement along the surface. As we cannot hope to study all this in a single investigation, we first study how V depends on d, keeping F and θ constant. The result is that if d is doubled the petrol consumption is doubled, if d is trebled V is trebled. In general

$$V \propto d, \text{ keeping } F \text{ and } \theta \text{ constant} \ldots\ldots\ldots (9)$$

—a result which should be familiar to any car owner.

Next we arrange that it requires, say, twice the pull to move the load, perhaps by roughening the ground, and measure the petrol consumption. Keeping the same d and θ as before, we change the required pulling force and repeat the experiment with these different values of F. The result:

$$V \propto F, \text{ keeping } d \text{ and } \theta \text{ constant} \ldots\ldots\ldots (10)$$

—again a result which should hardly surprise us.

Lastly, we keep F and d constant and vary θ. We discover that

$$V \propto \cos\theta, \text{ keeping } F \text{ and } d \text{ constant*} \ldots\ldots\ldots (11)$$

It is certainly true that in the case when $\theta = 90°$, i.e. $\cos\theta = 0$, the load will not move along the surface and so no fuel can be used in doing this particular job.

Combine the result of the three separate experiments, equations 9, 10 and 11 into a single relation.

* *If you have forgotten what cos θ means, refer to* MAFS, *section* 4.*A*.1.

Combining equations 9, 10, and 11 gives:

$$V \propto Fd\cos\theta \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

the required expression relating the volume of petrol used with F, d, and θ. But the energy content of petrol must be proportional to the volume present. If it were not, we should find, for example, that a car would travel different distances on the second, third, fourth, etc., gallons of a tankful. We may therefore write:

The energy transferred from the petrol $\propto$ Volume V of petrol used.

Or, from equation 12:

The energy transferred from the petrol $\propto Fd\cos\theta$.

To complete the story, we must discover how the energy gained by the ground is related to the energy lost from the petrol. The two are actually proportional. When the petrol consumption is doubled, twice the number of useful jobs can be done on the now warm surface across which the load has been pulled. Useful jobs in this context might be boiling beakers of water, melting squares of jelly, even frying eggs on the surface. We therefore conclude that:

Energy transfer *from* the petrol *to* the ground $\propto Fd\cos\theta$ ..(13)

By convention the constant of proportionality is taken as unity, so

$$\text{Energy transfer} = Fd\cos\theta \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

In the SI system, the energy transfer will be measured in units of N m. However, for convenience, N m is shortened to joule (J). Equation 14 may be written somewhat differently by noting that $F\cos\theta$ is the component F_d of force F in the direction of movement of the load. (See section 4.D.7 of *MAFS*.)

joule

$$\text{i.e. Energy transfer} = F_d d \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

In the past it used to be fashionable to talk about 'doing work' when referring to energy transfers; the left-hand side of equation 15 would have been called the 'work done'. While there is nothing wrong with speaking of 'doing work', and we use the words ourselves, there are situations where 'doing work', with all its human connotations, just does not ring true. It is easier on the tongue to say, for example, that energy is transferred from chemical form in a battery to thermal form in a wire connected across the terminals than to say that the battery *does work* on the wire.

SAQ 8

Make an order of magnitude calculation of the (chemical) energy required to push a car by hand along a level road through a distance of 1 mile. Assume the car is put in 'neutral'. (As a clue, first try and decide, to the nearest power of ten, what force is required to keep a car moving.) Ignore the extra effort required to get the car going.

The problem is worked out on p. 47.

The answer is about 10^5 N m, i.e. 10^5 J.* This will be a minimum estimate of the energy required, for our bodies are nothing like a hundred per cent efficient at converting chemical energy to mechanical energy.

The next example not only provides further practice in calculating energy transfers; it demonstrates how the energy transferred when a force moves its point of application can be calculated graphically. This graphical technique will be employed in subsequent sections.

* *In the past* 4.18 *J used to be called* 1 *calorie so the answer might be written as* $10^5/4.18 = 2.4 \times 10^4$ *calorie [the Calorie (notice the capital letter) referred to in dietetics is* 1 000 *calories].*

calorie
Calorie

A 'black box' has a piece of string coming out of it. You pull on the string via a newton balance, and find that your pull varies with the length of the string pulled in the way shown in Figure 11. How much energy has been transferred from you to whatever is inside the box during the experiment?

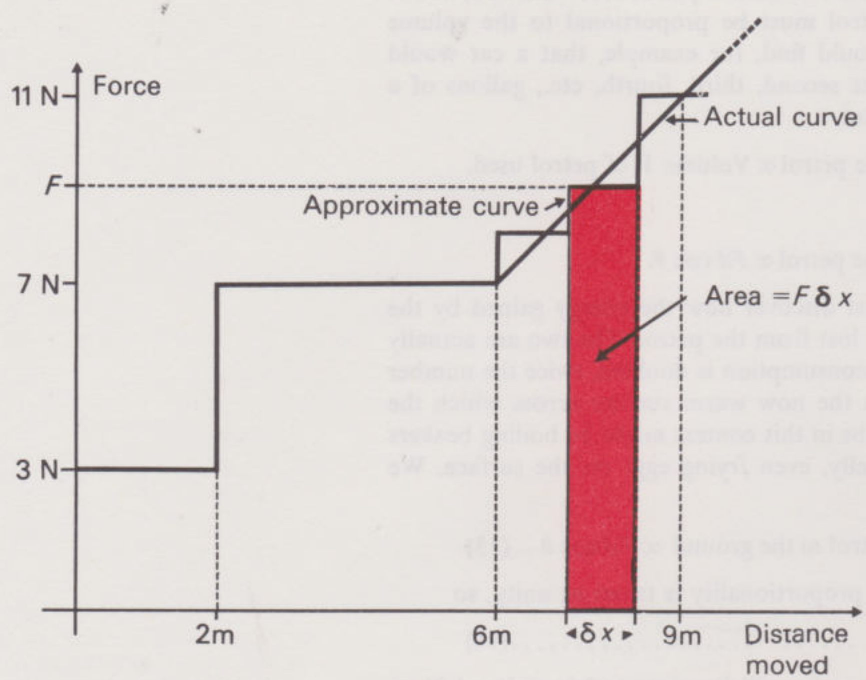


Figure 11 An example of an energy transfer. When a piece of string coming out of a black box was pulled, the pulling force varied with distance pulled in the manner shown.

Between 0 and 2 m the energy transferred is, from equation 15, equal to 3 N × 2 m = 6 N m = 6 J, which you should note is also equal to the area under this portion of the force-distance curve. From 2 to 6 m the energy transferred is 7 N × 4 m = 28 J; again the area under this section of the force-distance curve. To evaluate the energy transferred in the section from 6 m up to 9 m, we approximate the changing force as closely as we wish by a succession of steps. The energy transferred in pulling the string through a distance δx is $F\delta x$, where F is the force required to pull the string. This product $F\delta x$ is the area under this step, and is shown shaded in Figure 11. Repeat the argument under each step, add up all the areas and you will have proved that the total energy transferred between 6 and 9 m is the area under this section of the plot. Indeed the total energy transferred throughout the experiment is the total area under the force-distance curve. The area under the curve between 6 and 9 m is equal to the area of the rectangle of height 7 N and of length 3 m, plus the area of the triangle, of height 4 N and of base length 3 m;* i.e. the energy transferred between 6 and 9 m is 21 J + 6 J. Hence the total energy transferred from the puller into the box is 6 J + 28 J + 27 J = 61 J.

4.4.3 Potential energy

Some types of energy transfer occur so frequently that it is worthwhile making once-for-all calculations. A particularly important example is the energy required to bring one body up from infinity to a point at a given distance from another body which attracts or repels it in a well-defined manner. When the body arrives at this point, it is said to have potential energy (P.E.) which is the energy required to bring the body up from infinity to the point. The potential energy is labelled as positive if the 'pusher' must provide energy in bringing the body up, and negative if energy is transferred to the 'pusher' in the process. The concept of potential energy is not a simple one, so you should not be unduly worried if it eludes you on a first reading.

potential energy

* *The area of a triangle is $\frac{1}{2}$ (base × height). See* MAFS, *section 2.A.2.*

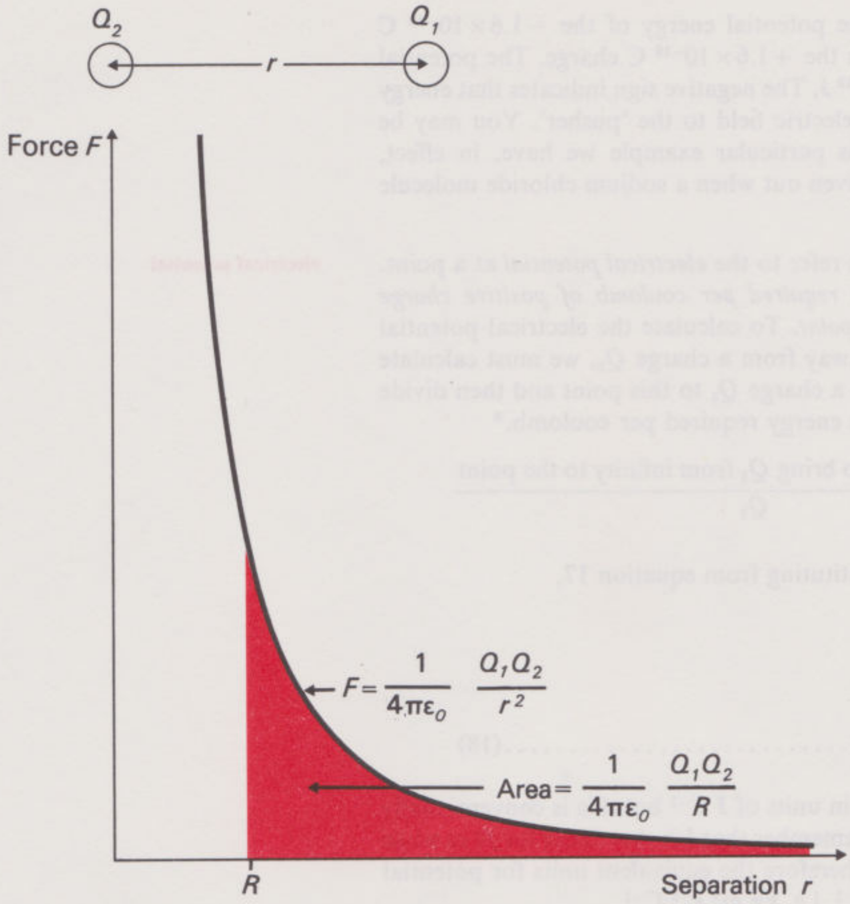


Figure 12 Electrostatic potential energy. The shaded area represents the energy required to bring Q_1 up from infinity to within a distance R of Q_2, i.e. it represents the potential energy of Q_1.

The two spheres of Figure 12 carry charges Q_1 and Q_2 respectively. As Q_1 approaches Q_2 the magnitude of the force F which Q_1 exerts on Q_2, when they are separated by a distance r, is given by equation 5 namely $F = Q_1 Q_2/4\pi\epsilon_0 r^2$. The variation of F with r is shown plotted in Figure 12. From what we have just learnt in the worked example of section 4.2.2, the total energy required to bring Q_1 from infinity up to a distance R from Q_2 is the area under the force-distance curve between R and infinity, and this is shown shaded in Figure 12. While anyone familiar with elementary integral calculus may be able to work out the result for themselves (a derivation is given in Appendix 1 (Black)), we can at this stage only state that

the area under the curve between R and infinity $= \dfrac{Q_1 Q_2}{4\pi\epsilon_0 R}$ ....(16)

and consequently

the P.E. of Q_1 when a distance R from Q_2 $= \dfrac{Q_1 Q_2}{4\pi\epsilon_0 R}$ ....(17)

Note, and this is important, that the denominator of equation 17 contains R, not R^2.

Satisfy yourself that the units of the right-hand side of equation 16 or 17 are those of energy.

In the SI system, Q_1 and Q_2 are in C, $1/4\pi\epsilon_0$ is in N m^2 C^{-2} (see p. 19), and R is in m. Therefore the right-hand side of equation 17 will be in C^2 N m^2 C^{-2} m^{-1}, i.e. N m or J, which is indeed the SI unit of energy.

SAQ 9
What energy is required to bring a charge of -1.6×10^{-19} C up from infinity to a distance of 2.5×10^{-10} m from a charge of $+1.6 \times 10^{-19}$ C?

The answer is worked out on p. 47.

By definition, this energy is the potential energy of the -1.6×10^{-19} C charge when 2.5×10^{-10} m from the $+1.6 \times 10^{-19}$ C charge. The potential energy works out as -9.2×10^{-19} J. The negative sign indicates that energy has been transferred from the electric field to the 'pusher'. You may be interested to know that in this particular example we have, in effect, estimated the energy which is given out when a sodium chloride molecule is formed.

You will frequently hear people refer to the *electrical potential* at a point. By this they mean the *energy required per coulomb of positive charge brought up from infinity to the point.* To calculate the electrical potential $V(R)$ at a point a distance R away from a charge Q_2, we must calculate the energy required to bring up a charge Q_1 to this point and then divide the answer by Q_1 to obtain the energy required per coulomb.*

electrical potential

i.e. $$V(R) = \frac{\text{Energy required to bring } Q_1 \text{ from infinity to the point}}{Q_1}$$

i.e. $V(R) = \dfrac{\text{P.E. at } R}{Q_1}$ or, substituting from equation 17,

$$V(R) = \frac{Q_1 Q_2}{4\pi\epsilon_0 R} \bigg/ Q_1$$

i.e. $$V(R) = \frac{Q_2}{4\pi\epsilon_0 R} \quad \dots\dots\dots (18)$$

Electrical potential is measured in units of J C^{-1} but this is conventionally shortened to *volt* (V). You will remember that J is shorthand for N m while N is shorthand for kg m s^{-2}. Therefore the equivalent units for potential are N m C^{-1} or (kg m s^{-2}) m C^{-1}, i.e. kg m^2 s^{-2} C^{-1}.

volt

Another term you will often hear used is the electrical *potential difference* (P.D.) between two points. By this is meant the energy required per coulomb of positive charge moved from one point to the other. If, as in Figure 13, a charge Q_1 is brought up from a distance of, say, R_2 from Q_2 to a distance R_1 from Q_2, the energy required is the shaded area. But this area is only the area under the curve from R_1 to infinity minus the area under the curve from R_2 to infinity. Both of these areas we know from equation 18. Therefore,

potential difference

energy required in bringing Q_1 from R_2 to R_1 $$= \frac{Q_1 Q_2}{4\pi\epsilon_0 R_1} - \frac{Q_1 Q_2}{4\pi\epsilon_0 R_2}$$

Dividing through by Q_1 gives:

energy required per unit positive charge brought from R_2 to R_1 $$= \frac{Q_2}{4\pi\epsilon_0 R_1} - \frac{Q_2}{4\pi\epsilon_0 R_2}$$

The left-hand side is, by definition, the potential difference between the two points,

i.e. potential difference between the points $= V(R_1) - V(R_2) \dots\dots (19)$

The following example shows the usefulness of this apparently abstract calculation.

A 'battery' has terminals marked O, +1.5 V, +3.0 V, +4.5 V, +6.0 V, +7.5 V. How much energy is given out if +1 C of charge (i.e. 6.7×10^{18} electrons) is allowed to move through a wire connecting the 7.5 V terminal to the 1.5 V one?

* *Note that* V (R) *does* not *mean* 'V *times* R'. *We call the electrical potential* V (R) *instead of just* V, *to remind ourselves that it depends on* R.

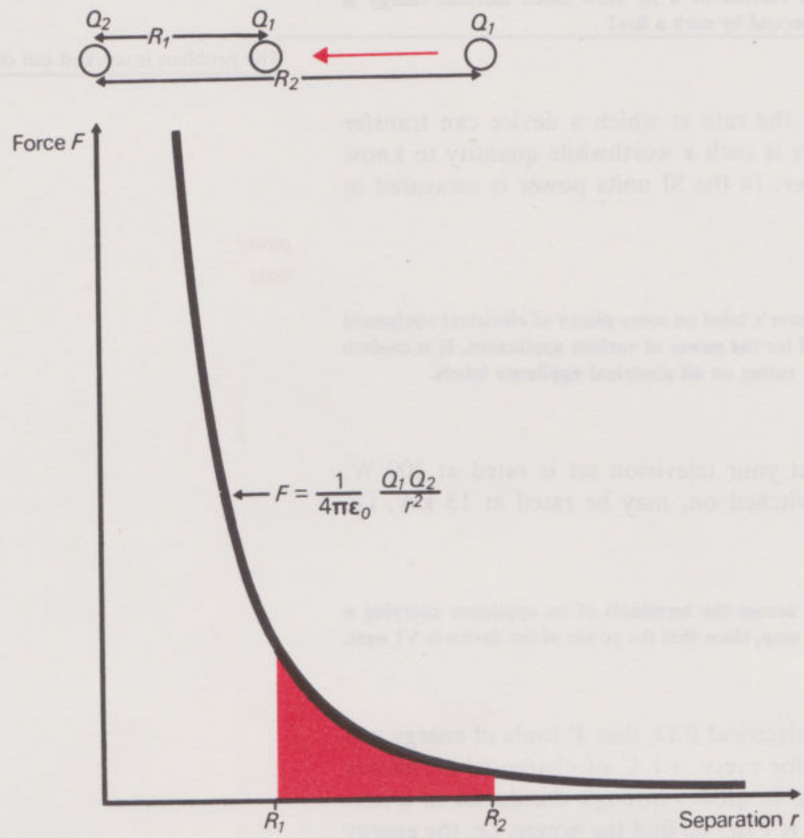


Figure 13 *Illustrating the electrical potential difference between two parts. The shaded area represents the energy required to move a charge* Q_1 *from a distance* R_2 *to a distance* R_1 *from a charge* Q_2.

The potential difference between these two terminals is, from equation 19, $7.5\ V - 1.5\ V = 6\ V = 6\ J\ C^{-1}$. In other words, 6 J of energy is transferred to the wire (as heat) when 1 C of charge flows between these terminals. As the difference between any two adjacent terminals on this battery is 1.5 V we can obtain 1.5 J of heat energy for every coulomb transferred between two adjacent terminals.

An electron (charge -1.6×10^{-19} C) is accelerated from rest through a potential difference of 50 V. How much energy has it acquired in the process?

As the potential difference is the energy transferred per coulomb, the energy acquired by the electron is the potential difference × charge; i.e. $50\ V \times (1.6 \times 10^{-19})\ C$, i.e. $50\ J\ C^{-1} \times (1.6 \times 10^{-19})\ C = 8.0 \times 10^{-18}$ J. If one electron is accelerated through a P.D. of 1 V, it acquires an energy of $1 \times (1.6 \times 10^{-19})$ J. Many scientists use the term *electron volt* as an energy unit, by which they mean the energy acquired by one electron in moving through a potential difference of 1 V, rather as we have used N m or J as our unit of energy. Since 1.6×10^{-19} J is a rather small unit, even for nuclear physicists, they may for example use the term MeV which is the energy required by an electron in moving through a P.D. of 10^6 V. (1 MeV= 1 Mega—electron volt.)

electron volt

MeV

What, in joules, is the value of 1 MeV?

The answer is $10^6 \times (1.6 \times 10^{-19})\ J = 1.6 \times 10^{-13}$ J. You are not expected to remember these numbers, for you can always look them up in section 3 of *HED*.

SAQ **10**

The potential difference between the 'live' and the 'neutral' terminals on a mains plug is typically 250 V in Britain. When connected to such mains a one-bar electric fire takes a current of 4 A. How much thermal energy is transferred to the room per second by such a fire?

The problem is worked out on p. 47.

The answer is 10^3 J s^{-1}. Indeed the rate at which a device can transfer energy from one form to another is such a worthwhile quantity to know that it is given the name of *power*. In the SI units power is measured in J s^{-1}, abbreviated to *watt* (W).

power

watt

Have a look at the manufacturer's label on some pieces of electrical equipment about your home to get a feel for the power of various appliances. It is modern practice to include the power rating on all electrical appliance labels.

You may find, for example, that your television set is rated at 200 W. Your cooker with everything switched on, may be rated at 13 kW, i.e. 13 000 W.

If there is a P.D. of V **volt across the terminals of an appliance carrying a steady or 'direct' current of** I **amp, show that the power of the device is VI watt.**

It follows from the definition of electrical P.D. that V Joule of energy will be transferred to the appliance for every $+1$ C of charge which moves through it. Therefore if a charge δq moves through the device in a time interval δt the energy transferred is $V\delta q$. To find the power, i.e. the energy transferred per second, we must divide the energy transferred in a time δt by the time interval δt:

$$\text{Power} = \text{rate of transfer of energy}$$

$$= V\delta q/\delta t$$

Since a charge δq moves through the device in a time δt, $\delta q/\delta t$ is the charge flowing through the appliance per second, i.e. it is the current I.

$$\therefore \quad \text{Power} = VI \quad \ldots\ldots\ldots\ldots\ldots (20)$$

Equation 20 can only be applied to 'direct current' devices. It cannot, in general, be applied to calculate the power of an 'alternating current' device.*

SAQ **11**

What current passes through a 150 W light bulb operating from a 250 V direct current supply?

The problem is worked out on p. 47.

The answer is 0.6 A.

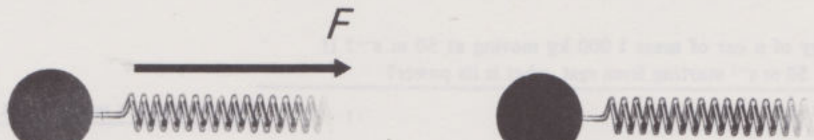
4.4.4 Kinetic energy

Another calculation which is worth making once and for all is the energy transferred to an object as it is accelerated from rest. The puck of mass m in Figure 14a is acted on by a constant force F through a distance s, the force being in the same direction as the displacement s. In the process

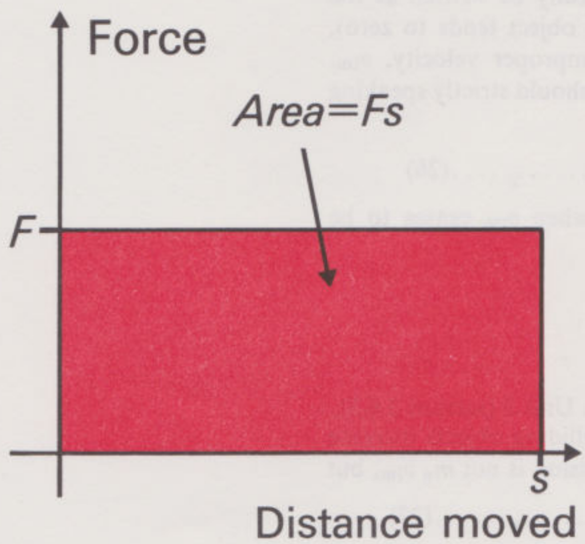
* *Equation 20 is however applicable to alternating current devices which are purely 'resistive', like electric fires.*

an energy Fs (see equation 15) is transferred to the puck; we say the puck acquires a kinetic energy (written K.E.), i.e. energy of motion, of Fs. This energy is also the area under the force-distance curve, as shown in Figure 14b. We can express the energy transfer differently by recalling our primary definition of a force (equation 14 of Unit 3), namely

$$F = ma \quad \text{..................(21)}$$



(a)



(b)

Figure 14 *Kinetic energy. (a) A puck of mass* m *is acted on by a force* F *through a distance* s. *(b) The energy transfer is shown as the shaded area, of magnitude* Fs.

So the K.E. acquired by the puck is given by

$$\begin{aligned} \text{K.E.} &= \text{Area under force-distance curve up to distance } s \\ &= Fs \\ &= ma\,s \end{aligned}$$

i.e.

$$\text{K.E.} = m(as) \quad \text{................(22)}$$

But, under conditions of constant acceleration, there is a simple relation between the final velocity v, the initial velocity u, the acceleration a, and the distance gone, s.

Do you remember the relation?

If not, you should refer to Appendix 1, Unit 3; in particular see equation 7:

$$v^2 = u^2 + 2\,as \quad \text{.............(23)}$$

Since the puck started from rest, i.e. $u = 0$,

$$v^2 = 2\,as \quad \text{.................(24)}$$

or

$$as = \tfrac{1}{2}v^2$$

Substituting equation 24 into equation 22 gives

$$\text{K.E.} = \tfrac{1}{2}mv^2 \quad \text{................(25)}$$

Why, you may ask, did we go to all this trouble when we could simply have multiplied the force by the distance? In some situations this may indeed be possible, but if a moving object of mass m emerges from hiding with a velocity v, we have no idea what force acted on it nor through what distance it acted; yet we can determine its kinetic energy by applying equation 25.

***SAQ* 12**

What is the kinetic energy of a car of mass 1 000 kg moving at 50 m s^{-1}? If the car took 20 s to reach 50 m s^{-1} starting from rest, what is its power?

The problem is worked out on p. 48.

The kinetic energy is 1.25×10^6 J and the car had an average power of 6.25×10^4 W, or 62.5 kW.*

It should perhaps be pointed out that all our discussions of energy transfers in mechanical systems have been non-relativistic, i.e. the objects have all been moving at speeds which are very much less than that of light. Thus, in equation 25, the mass m should really be written as the rest mass, m_0 (the mass when the velocity of the object tends to zero), while the measured velocity, v, is actually the improper velocity, v_{im}. Therefore the low-speed relation for kinetic energy should strictly speaking be written as

$$\text{K.E.} = \tfrac{1}{2}\, m_0\, v_{\text{im}}^2 \quad \ldots\ldots\ldots\ldots(26)$$

But what, if anything, happens to equation 26 when v_{im} ceases to be negligible compared with the velocity of light?

4.4.5 Relativistic energy

While discussing the conservation of momentum in Unit 3 (section 3.6.3), it was pointed out that, as the velocities of the colliding objects increase towards that of light, what is conserved in the collision is not $m_0\, v_{\text{im}}$, but

$$p = m_0\, v_{\text{im}} / \sqrt{1 - v^2_{\text{im}}/c^2} \quad \ldots\ldots\ldots\ldots(27)$$

It was also pointed out that one way of interpreting equation 27 is to say that instead of remaining constant at m_0 the mass m of an object changes with speed v_{im} as follows:

$$m = m_0 / \sqrt{1 - v^2_{\text{im}}/c^2}$$

The mass m, you may remember, is called the relativistic mass of the object (section 3.6.3 of Unit 3). We might therefore be tempted to the conclusion that the relativistic expression for kinetic energy should be, not $\frac{1}{2}\, m_0\, v^2_{\text{im}}$, but

$$\tfrac{1}{2}\left(\frac{m_0}{\sqrt{1 - v^2_{\text{im}}/c^2}}\right) v^2_{\text{im}}.$$

Such a conclusion would, however, be incorrect. To arrive at the correct expression for the kinetic energy of an object we must go back to first principles.

Can you remember how we derived the expression $\frac{1}{2}\, mv^2$, *i.e.* $\frac{1}{2}\, m_0 v^2_{\text{im}}$?

We multiplied the force F which acted on the object by the distance s through which it acted; this being the fundamental definition for the energy transferred to the object (equation 15), i.e. we evaluated the area

* *A unit of power that is sometimes used is the* horse-power. 1 *horse-power* = 746 *W* = 0.746 *kW. It is evidently not an approved SI unit! The car in the example of* SAQ 12 *had a power of* $6.25 \times 10^4 / 746 = 84$ *horse-power.*

horse-power

under the graph of F plotted against s. We then assumed, and this is the point to note, that

$$F=ma \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

To spell it out, and it is worth spelling out, we assumed that if the body of mass m has an acceleration a at a particular instant, then the force acting on the body at that instant is the product of m and a. While this may appear to be a simple enough definition of the magnitude of the force F, equation 21 as it stands is a useless definition of force when the speed of the object approaches that of light; useless because it does not state whether the mass m is the value assigned by a stationary observer or by one travelling with the object. Equation 21 is also useless because it does not state the positions of the clocks used in measuring the speeds of the object and hence the rate at which that speed is changing. In other words, a worthwhile definition of force must state quite explicitly where all the observers (actually the clocks) are to be stationed.

It is perhaps interesting that the more general definition of force, the definition that may be applied unambiguously even for objects moving at speeds approaching that of light, can be traced back to Newton. To see how it is arrived at, we first rewrite equation 21 in a somewhat different form as follows. This re-arrangement will be carried out in the low-speed limit where the questions about where the clocks are located do not arise.

From equation 21

$$F=ma \quad \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

But from the definition of acceleration as the rate of change of velocity

$$a=\delta v/\delta t$$

where δv is the change in velocity that takes place in a time δt.

I.e.

$$F=m\,\delta v/\delta t \quad \ldots\ldots\ldots\ldots\ldots (28)$$

Assuming tentatively that m is constant, $m\delta v$ just means the change in the value of mv. Since mv is the momentum p of the object, $m\delta v$ just measures δp, the change in the value of p*.

Substituting δp for $m\delta v$ in equation 28 gives

$$F=\delta p/\delta t \quad \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

Equation 29 was actually Newton's own definition of force, namely the change δp in the momentum of a body divided by the time interval δt during which this momentum change occurred. Since Newton's clocks were fixed in the laboratory the time interval δt in equation 29 is, in our terminology, an improper time interval δt_{im}. Therefore his definition of a force was

$$F=\delta p/\delta t_{\mathrm{im}} \quad \ldots\ldots\ldots\ldots\ldots (30)$$

To Newton, p was simply mv (or $m_0 v_{\mathrm{im}}$ in our notation). To us the correct definition of p is always

$$p=m_0 v_{\mathrm{im}}/\sqrt{1-v^2_{\mathrm{im}}/c^2} \ldots\ldots (31)$$

Taken together, equations 30 and 31 provide a working definition of the force acting on a body. You will not be expected to remember these equations but, given equations 30 and 31, you may be able to describe how they could be employed to calculate the force acting on a body.

* *As an example suppose the velocity of a 2 kg puck changes from 11.0 m s^{-1} to 11.7 m s^{-1}. The change in momentum δp is the final momentum minus the initial momentum, i.e. (2 × 11.7) kg m s^{-1} − (2 × 11.0) kg m s^{-1} = 23.4 kg m s^{-1} − 22.0 kg m s^{-1} = 1.4 kg m s^{-1}.) The corresponding value of* m δv *is* 2(11.7 − 11.0) *kg m s^{-1} = 1.4 kg m s^{-1}, which is identical to* δp.

You are in a 'laboratory' situated somewhere in outer space. A space ship 'burns' past your laboratory. How would you set about deducing the force being provided by the ship's engines? Assume that the passengers on the space ship have told you their assessment of the mass m_0. Assume also that m_0 remains constant throughout (i.e. that the total mass of the ship is very much greater than that of the fuel burnt in the engine).

You must measure the momentum (as defined by equation 31) at one instant and then measure it again a time δt_{im} later. Dividing the change in the momentum δp by the time interval δt_{im} gives the required force (equation 30). To determine the momentum you must, as equation 31 shows, determine the improper speed of the ship; this involves placing a couple of clocks a known distance apart in the 'laboratory'. Dividing the separation between the clocks by the difference in their time readings taken as the ship passes each clock gives the improper velocity.

How would you, situated in your laboratory, calculate the energy which has been transferred from chemical form in the fuel into kinetic energy, while the rocket moves through a distance s as measured in your laboratory? (Assume s to be in the same direction as that in which the rocket moves.)

As we have seen (equation 15), the energy transferred is defined as being 'force × distance' or equivalently as being the area under the appropriate section of the force-distance curve. To calculate the energy transfer, you would have to calculate F as before, using equations 30 and 31. Then you would plot a graph showing how this calculated value of F varied as the ship moved through the distance s measured in the laboratory. Finally, you would evaluate, perhaps by 'counting squares' the area under the curve covering the distance s. While such a calculation can always be made from first principles each time it is required, it is convenient to have a once-for-all expression.

You saw, in section 4.4.4, that it is easy to work out the kinetic energy, $\frac{1}{2}m_0v_{\text{im}}^2$, when the body is moving slowly enough for the simple formula $F=ma$ to apply.

The calculation is more difficult, however, when force is defined by equations 30 and 31. If you are interested, and know a little calculus, you will find the calculation worked through in Appendix 2 (Black). We shall just quote the result here:

$$\text{K.E.} = \gamma m_0c^2 - m_0c^2 \quad \ldots\ldots\ldots(32)$$

where

$$\gamma = 1/\sqrt{1-v^2_{\text{im}}/c^2} \quad \ldots\ldots\ldots(33)$$

The symbol γ (pronounced 'gamma') is introduced simply to save the trouble of writing down $1/\sqrt{1-v^2_{\text{im}}/c^2}$ which, as you may have noticed, is a quantity that keeps cropping up whenever we have to do with relativistic motion, that is with motion in which the improper velocity, v_{im}, is not negligible compared with the velocity of light, c.

Note, in passing, that equation 31 in Unit 3 (section 3.6.3) can be written in a conveniently short form, using γ:

$$m = m_0/\sqrt{1-v^2_{\text{im}}/c^2} \quad \ldots\ldots(31) \text{ (Unit 3)}$$

$$= \gamma m_0 \quad \ldots\ldots\ldots\ldots\ldots(34)$$

So the kinetic energy could also be written as

$$\text{K.E.} = mc^2 - m_0c^2 \quad \ldots\ldots\ldots\ldots(35)$$

in which we have replaced γm_0 by m.

Before we discuss further the implications of the result expressed in equations 32 or 35, it would be as well to check whether this formula for

the kinetic energy can be reduced to the more familiar formula $\frac{1}{2}m_0v^2_{\mathrm{im}}$ when $v_{\mathrm{im}} \ll c$.

This amounts to finding an approximate value for

$$\gamma = 1/\sqrt{1 - v^2_{\mathrm{im}}/c^2} = (1 - v^2_{\mathrm{im}}/c^2)^{-\frac{1}{2}}$$

when $$v_{\mathrm{im}} \ll c.$$

You have come across this sort of problem before—refer back to Unit 3, section 3.3.1, equation 3, to refresh your memory. Here it is again:

$$(1+x)^m \approx 1 + mx + \quad \ldots\ldots\ldots\ldots\ldots (3)$$

(Unit 3)

provided that $x \ll 1$.

In the present case we have $x = -v^2_{\mathrm{im}}/c^2$, which is indeed very much smaller than unity if $v_{\mathrm{im}} \ll c$, and we have $m = -\frac{1}{2}$.

So $$(1 - v^2_{\mathrm{im}}/c^2)^{-\frac{1}{2}} = 1 + \tfrac{1}{2}(v^2_{\mathrm{im}}/c^2) + \ldots$$

Put this approximate value for γ into equation 32:

$$\begin{aligned} \text{K.E.} &\approx (1 + \tfrac{1}{2}v^2_{\mathrm{im}}/c^2)m_0c^2 - m_0c^2 \\ &\approx \tfrac{1}{2}v^2_{\mathrm{im}}/c^2 \,.\, m_0c^2 \\ &\approx \tfrac{1}{2}m_0v^2_{\mathrm{im}} \end{aligned}$$

So the usual, non-relativistic formula is an approximate form of the more general, relativistic formula (equation 32). The approximation is a very good one at low velocities.

Now look again at equation 35.

Equation 35 tells us that kinetic energy—energy associated with motion relative to the observer—is made up of some amount of energy, (mc^2), minus some other amount of energy (m_0c^2). Let's turn equation 35 around a bit.

$$mc^2 = m_0c^2 + \text{K.E.} \quad \ldots\ldots\ldots\ldots\ldots (36)$$

This tells us that some amount of energy (mc^2) is made up of some other amount of energy (m_0c^2), which does *not* depend on the motion of the object, plus the kinetic energy which *does* depend on the motion of the object—it is zero when the velocity v_{im} is zero.

This suggests strongly that the quantity mc^2 must be some sort of 'total energy', and that its two components are a 'rest-mass energy', m_0c^2, and a kinetic energy:

rest-mass energy

'total (relativistic) energy' = 'rest-mass energy' + (relativistic) kinetic energy

$$mc^2 = m_0c^2 + \text{K.E.} \quad \ldots\ldots\ldots\ldots\ldots (36)$$

Now look again at equation 34:

$$m = \gamma m_0$$

This tells us that when $v_{\mathrm{im}} = 0$, which makes $\gamma = 1$, then $m = m_0$; to measure m, the 'rest-mass' we have to measure the mass of the object when it is at rest, or at least when it is moving so slowly that we can neglect v_{im} compared with c.

There is no difficulty about measuring the mass of a stationary, or quasi-stationary object, at least in principle. So we can, in principle, say what the 'rest-mass energy', m_0c^2 is. If, then, we measure its improper velocity, v_{im}, we can work out its kinetic energy, using equations 32 and 33. And so we can work out the 'total energy', mc^2.

All this seems nice and neat and plausible, but does the quantity 'rest-mass energy' have any physical meaning?

It was Einstein who first guessed that if, in some process, matter of rest-mass m_0 were to 'disappear', the equivalent energy m_0c^2 should appear in its place.

The name of Hiroshima is a reminder that Einstein's guess was correct.

It is a small step, but conceptually a very important one, from saying

> 'in certain processes mass can get converted into an equivalent amount of energy',

to saying

> 'mass and energy are equivalent—they are two equivalent forms or modifications of a single attribute of matter'.

The latter is the point of view of contemporary physics. It is all the more firmly held because the disappearance or appearance of matter (energy) and the corresponding appearance or disappearance of the corresponding energy (mass) turns out to be an extremely commonplace occurrence.

You have already encountered one example of such a process in the TV programme of Unit 2, where you saw that muons disintegrate spontaneously and shoot out positrons in the process. The positrons get their energy (both kinetic energy and rest-mass energy) from the conversion of the rest-mass of the muon.

Indeed in *all* processes, physical and chemical, the appearance of energy is always accompanied by the corresponding disappearance of the equivalent amount of mass. But, unless the amount of energy that appears or disappears is quite large, the equivalent amount of mass is too small to be measurable; after all, c^2 is a very large number: 9×10^{16} m^2 s^{-2}.

To get an idea of the quantitative implications of m_0c^2, try the following problems:

The rest-mass of the debris after an atomic bomb explosion has been estimated to be some 10^{-3} kg less than the initial rest-mass of the materials. How much energy is released in such an explosion?

The energy released is the rest-mass 'destroyed' $\times c^2$, or $10^{-3} \times 9 \times 10^{16}$ kg m^2 s^{-2}

$= 9 \times 10^{13}$ J (since 1 J = 1 kg m^2 s^{-2}).

Most of the energy will be released as heat, which, as you will learn in Unit 5, is none other than the kinetic energy of the atoms and molecules of all the matter in the neighbourhood of the explosion.

Probably the amount of energy 9×10^{13} J doesn't mean anything to you. An order of magnitude estimate in terms of roast chickens might help.

If a 15 kW domestic cooker takes 2 hours to roast a chicken, make an order of magnitude estimate of how many chickens might be roasted in the heat of such an atomic explosion.

Assuming the cooker is on continuously, it will be using 15 000 J of energy every second (15 kW = 15 000 W and a watt is a joule per second).

So in 2 hours, the cooker uses $15 \times 10^3 \times 2 \times 60 \times 60$ J, or about 10^8 J. Since the energy released in the explosion is about 10^{14} J, it follows that the number of chickens that could be roasted with that amount of energy is $10^{14}/10^8 = 10^6$—a 'mega chicken'.

This vast amount of energy derived from a mass loss of only 1 gm.

It follows, of course, that the relatively modest amounts of energy given out in ordinary chemical reactions, of the kind you will be doing in your home experiments—a few joules at most—involve negligible mass changes. This is because chemical reactions depend on electromagnetic forces which are very much weaker than nuclear forces, and so the energies liberated in chemical reactions are characteristically some millions of times less than those liberated in nuclear reactions.

4.5 Recapitulation

Surveying the basic forces, in terms of which all macroscopic forces can be explained, we looked in turn at gravitational, electromagnetic and nuclear interactions. We saw how the force F of gravitational attraction between spherical masses depended on the value of the masses m_1 and m_2 and on their separation r apart. In particular we discovered that (equation 4)

$$F = Gm_1m_2/r^2$$

where G is determined experimentally.

Having found that when electric currents flow in adjacent metallic wires the two wires interact, we used this phenomenon as a means of defining the magnitude of the current. We then described an experiment in which known amounts of electric charge could be collected on metal spheres. A study of how the electrostatic force F between two spheres possessing charges Q_1 and Q_2 depended on their separation r apart showed that (equation 5)

$$F = \left(\frac{1}{4\pi\epsilon_0}\right) Q_1\, Q_2/r^2$$

where $1/4\pi\epsilon_0$ is determined experimentally.

Although we discussed electromagnetic forces in terms of magnetic and electrostatic forces, we pointed out that both are due to electric charges, the magnetic force arises when the charges move relative to the observer; electrostatic forces arise when the charges are at rest relative to the observer. It was illustrated how the conceptual difficulty of 'action at a distance' could be avoided if fields were introduced. As an important example of a field, the electric field at a point distant r from a charge Q is (equation 7)

$$E \doteq \left(\frac{1}{4\pi\epsilon_0}\right) Q/r^2$$

On looking into what happened whenever a force moved its point of application, we were led to think about the role of fuels and so to the concept of energy. We saw that a sensible measure of the energy transferred in a mechanical process is given by the area under the graph showing how the applied force varies with distance. This result was then applied by calculating potential energies, a particularly important example of which is the potential energy P.E. of a charge Q_1 at a distance R from another charge Q_2. It was shown (equation 17) that

$$\text{P.E.} = \left(\frac{1}{4\pi\epsilon_0}\right) Q_1\, Q_2/r$$

The kinetic energy of a body was shown to be $\frac{1}{2}m_0 v_{\text{lm}}^2$ so long as $v_{\text{lm}} \ll c$. Here the rest-mass, m_0, is mass determined at low speeds. Examining the general expression for the kinetic energy of an object, we found that the mass of an object increases as its kinetic energy increased. This expression also suggested that an amount of energy m_0c^2 would appear if a mass m_0 of matter were to disappear.

This led us to the idea of equivalence of mass and energy.

Appendix 1

Electrical potential energy

The area under the curve shown in Figure 12 (p. 29) between R and infinity represents the electrical potential energy (P.E.) of Q_1 when it is a distance R from Q_2; the energy transferred in a process being the area under the appropriate section of the force-distance curve.

i.e. P.E. = Area under the force-distance curve between infinity and R

$$= \int_R^\infty F \, dr$$

But from equation 5, $F = Q_1 Q_2 / 4\pi\epsilon_0 r^2$.

Therefore:

$$\text{P.E.} = \int_R^\infty \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} \, dr$$

Taking the constant terms Q_1, Q_2, and $1/4\pi\epsilon_0$ outside the integral gives

$$\text{P.E.} = \frac{Q_1 Q_2}{4\pi\epsilon_0} \int_R^\infty \frac{dr}{r^2}$$

$$= \frac{Q_1 Q_2}{4\pi\epsilon_0} \left[-\frac{1}{r} \right]_R^\infty$$

$$\therefore \quad \text{P.E.} = \frac{Q_1 Q_2}{4\pi\epsilon_0} \left[0 - \left(-\frac{1}{R} \right) \right]$$

$$\text{i.e.} \quad \text{P.E.} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R}$$

Appendix 2 (Black)

Relativistic kinetic energy

As a result of the discussion in section 4.4.5, we agreed to define the force F acting on a body of rest-mass m_0, moving at a speed v_{im}, as

$$F=\frac{\mathrm{d}p}{\mathrm{d}t_{\mathrm{im}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(37)$$

where

$$p=\frac{m_0 v_{\mathrm{im}}}{\sqrt{1-(v^2_{\mathrm{im}}/c^2)}} \quad \ldots\ldots\ldots\ldots(31)$$

Equation 37 is equation 30 taken to the limit as δt_{im} tends to zero.

To calculate the energy transferred to a body of rest-mass m_0 as it is accelerated from rest to a final velocity v_{fm}, corresponding to a final momentum p, we must calculate the appropriate area under a graph showing how the force acting on the body changes with distance, x, as measured by a stationary observer. Using the notation of calculus, the resulting energy of the body, its kinetic energy after it has moved through a distance, x, may be written as

$$\text{K.E.}=\int_0^x F\,\mathrm{d}x$$

where F is defined by equations 37 and 31.

i.e.
$$\text{K.E.}=\int_0^x \frac{\mathrm{d}p}{\mathrm{d}t_{\mathrm{im}}}\,\mathrm{d}x$$

Rewriting the term inside the integral as $\mathrm{d}p\,(\mathrm{d}x/\mathrm{d}t_{\mathrm{im}})$ gives

$$\text{K.E.}=\int_0^p \mathrm{d}p\,\frac{\mathrm{d}x}{\mathrm{d}t_{\mathrm{im}}}$$

where p is the final momentum of the object.

But $\mathrm{d}x/\mathrm{d}t_{\mathrm{im}}=v_{\mathrm{im}}$

$$\therefore\quad \text{K.E.}=\int_0^p v_{\mathrm{im}}\,\mathrm{d}p$$

Integrating by parts, we obtain

$$\text{K.E.}=\Big[v_{\mathrm{im}}\,p\Big]_0^{v_{\mathrm{im}}}-\int_0^{v_{\mathrm{im}}} p\,\mathrm{d}v_{\mathrm{im}}$$

Substituting for p from equation 31, and also expressing $v\,\mathrm{d}v$ as $\frac{1}{2}\mathrm{d}\,(v^2)$, gives

$$\text{K.E.} = \left[\frac{m_0\, v_{\text{im}}^2}{\sqrt{1-(v^2_{\text{im}}/c^2)}}\right]_0^{v_{\text{im}}} - \frac{m_0}{2}\int_0^{v_{\text{im}}} \frac{\mathrm{d}(v^2)}{\sqrt{1-(v^2_{\text{im}}/c^2)}}$$

The second term is a standard form. Integrating it, we have

$$\text{K.E.} = \left[\frac{m_0\, v_{\text{im}}^2}{\sqrt{1-(v^2_{\text{im}}/c^2)}}\right]_0^{v_{\text{im}}} + \left[m_0\, c^2\sqrt{1-(v^2_{\text{im}}/c^2)}\right]_0^{v_{\text{im}}}$$

$$= m_0\, c^2\left[\frac{v_{\text{im}}^2/c^2}{\sqrt{1-(v^2_{\text{im}}/c^2)}} + \sqrt{1-(v^2_{\text{im}}/c^2)}\right]_0^{v_{\text{im}}}$$

$$= m_0\, c^2\left[\frac{1}{\sqrt{1-(v^2_{\text{im}}/c^2)}}\right]_0^{v_{\text{im}}}$$

$$\therefore\quad \text{K.E.} = \frac{m_0\, c^2}{\sqrt{1-(v^2_{\text{im}}/c^2)}} - m_0\, c^2$$

the general expression for the kinetic energy of a body of rest-mass m_0 moving at a velocity v_{im}.

Self-Assessment Questions

Section 4.2.1

Question 1 (*Objective 2*)

Make an order-of-magnitude estimate of the force of gravitational attraction between two adult human beings when standing side by side. Using a relation derived for spheres is bound to make the answer suspect; guesses of human masses correct to a factor of two or so will therefore be quite adequate.

Section 4.2.2

Question 2 (*Objective 3*)

Tick those statements below which correctly describe the nature of the electrodynamic interaction between two current-carrying wires.

1 The wires only interact if there is a current in both wires.
2 The wires will interact with a current in one wire alone.
3 The force between the wires is attractive when the currents are in opposite senses in each wire.
4 The force between the wires is attractive when the currents are in the same sense in each wire.
5 The forces decrease as the number of batteries in the circuits is increased.
6 The force acting on the wires is independent of the medium in which they are located.
7 The forces increase as the wires are brought closer together.

Question 3 (*Objective 4*)

Make an order-of-magnitude calculation of what would be the force of electrostatic attraction between two adults standing side by side if 1.5 C of charge had been transferred from one person to the other. Compare this electrostatic force with the force of gravitational attraction between the persons.

Question 4 (*Objective 5*)

The normal British one-bar electric fire takes a current of about 4 A. How many electrons pass through such a fire each hour?

Question 5 (*Objective 5*)

How many electrons were transferred from one person to the other in problem *SAQ* 3? Express this gain or loss as a percentage of the total number of electrons in each person. (You will need to know that atoms have an average diameter of about 10^{-10} m and that in living matter each atom contains an average of about 10 electrons.)

Section 4.3.2

Question 6 (*Objective 6*)

Calculate the electric field at a distance of 2×10^{-10} m from a charge of 1.6×10^{-19} C.

Question 7 (*Objective 6*)

What will be the force on a 1.6×10^{-19} C charge when introduced into a region where the electric field is 3.6×10^{10} N C^{-1}?

Section 4.4.2

Question 8 (*Objective 7*)

Make an order-of-magnitude calculation of the (chemical) energy required to push a car by hand through a distance of 1 mile. Assume the car is in 'neutral'. (As a clue, first try and decide, to the nearest power of ten, what force is required to keep a car moving.) Ignore the extra effort required to get the car going.

Section 4.4.3

Question 9 (*Objective 8*)

What energy is required to bring a charge of -1.6×10^{-19} C up from infinity to a distance of 2.5×10^{-10} m from a charge of 1.6×10^{-19} C?

Question 10 (*Objective 8*)

The potential difference between the 'live' and the neutral terminals on a mains plug is typically 250 V in Britain. When connected to such mains, a one-bar electric fire takes a current of 4 A. How much thermal energy is transferred to the room per second by such a fire?

Question 11 (*Objective 9*)

What current passes through a 150 W light bulb operating from a 250 V direct current supply?

Section 4.4.4

Question 12 (*Objective 8*)

What is the kinetic energy of a car of mass 1 000 kg moving at 50 m s^{-1}? If the car took 20 s to reach 50 m s^{-1} starting from rest, what is its power?

Self-Assessment Answers and Comments

Question 1

Assume the two adults have identical masses of 80 kg (i.e. about 13 stone) and that, when close together, their distance apart, between 'centres' will be about 0.3 m (i.e. about 1 ft.). Substituting $m_1 = m_2 = 80$ kg and $r = 0.3$ m into equation 4 (a relation, remember, which was derived using spheres) gives the force of attraction, F, as

$$F = G\, m_1 m_2/r^2$$

$$= \frac{6.67}{10^{11}} \times \frac{80^2}{0.3^2} \frac{\text{N m}^2}{\text{kg}^2} \frac{\text{kg}^2}{\text{m}^2}$$

substituting $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.

Because of the uncertainties inherent in applying equation 4 to non-spherical masses there would be little point, for example, in using 'log tables' to evaluate the answer—rough approximations will be quite adequate.

$$F \approx \frac{7}{10^{11}} \times \frac{6.5 \times 10^3 \text{ N}}{10^{-1}}$$

$$\approx \frac{4.5}{10^6}$$

$$F \approx 10^{-5} \text{ N}$$

The reason for deciding to approximate 4.5×10^{-6} N to 10^{-5} N, rather than to 10^{-6} N, is that the calculation has been made by multiplying and dividing quantities. To 'convert' 5×10^{-6} to 10^{-5} requires only a factor of 2 'wrong', but to convert it to 10^{-6} requires a factor of 5.

Question 2

The correct statements are 1, 4, 7. See section 4.2.2.

Question 3

Substituting $Q_1 = Q_2 = 1.5$ C and $r = 0.3$ m (say) into equation 5 gives the attractive force F as

$$F = \frac{Q_1\, Q_2}{4\pi\epsilon_0 r^2}$$

$$\approx \frac{9 \times 10^9 \times (1.5)^2}{(0.3)^2} \frac{\text{N m}^2}{\text{C}^2} \frac{\text{C}^2}{\text{m}^2}$$

$$\text{taking } \frac{1}{4\pi\epsilon_0} \approx 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

As equation 5 is really only applicable to spheres, rough and ready approximations can be made.

i.e.

$$F \approx \frac{10^{10} \times 2}{10^{-1}} \text{ N}$$

$$\approx 2 \times 10^{11} \text{ N}$$

$$\approx 10^{11} \text{ N}$$

Question 4

By 4 A is meant 4 C of charge per second passing a point in the wire. But there are $1/1.6\times10^{-19}=6.25\times10^{18}$ electrons per coulomb of charge (the electronic charge is 1.6×10^{-19} C).

i.e. $4\text{ A}=4\text{ C s}^{-1}$

$=4\text{ C}\times6.25\times10^{18}$ electrons s^{-1}

$4\text{ A}=2.5\times10^{19}$ electrons passing a point in the wire per second.

∴ In 1 hour the number of electrons passing through the fire will be $2.5\times10^{19}\times60\times60\approx10^{23}$.

Question 5

Since the electronic charge is 1.6×10^{-19} C there are $1/1.6\times10^{-19}=6.25\times10^{18}$ electrons per coulomb. Therefore the number of electrons in the 1.5 C charge, transferred from one person to another, is $1.5\times6.25\times10^{18}\approx10^{19}$.

To estimate the total number of electrons in a human body we estimate the number of atoms in the body and then multiply the result by 10; there is an average of about 10 electrons per atoms in living matter.

Treating an atom as a 10^{-10} m cube it will have a volume of $(10^{-10})^3$ m^3, i.e. 10^{-30} m^3. A human's volume is about 10^{-1} m^3 (e.g. 1.5 m high × 0.2 m wide × 0.3 m broad). Therefore the number of atoms in the body is about $10^{-1}/10^{-30}=10^{29}$. Hence the number of electrons is about $10\times10^{29}=10^{30}$. The gain or loss of the 10^{19} electrons expressed as a percentage of the total number of electrons in the body is

$$\frac{10^{19}}{10^{30}}\times100\text{ per cent}$$

$$=10^{-9}\text{ per cent}$$

Put differently only 1 atom in 10^{11} would have to lose or gain an electron to produce the total change in charge.

Question 6

The electric field E at a distance r from a charge q is, from equation 7,

$$E=q/4\pi\epsilon_0 r^2$$

Substituting $q=1.6\times10^{-19}$ C, $r=2\times10^{-10}$ m, and $1/4\pi\epsilon_0=9\times10^9$ N m^2 C^{-2} gives

$$E=\frac{9\times10^9\times1.6}{10^{19}\times(2\times10^{-10})^2}\ \frac{\text{N m}^2}{\text{C}^2}\ \frac{\text{C}}{\text{m}^2}$$

$$=\frac{9\times1.6\times10^{10}}{4}\text{ N C}^{-1}$$

$$=3.6\times10^{10}\text{ N C}^{-1}$$

Question 7

Since the electric field at a point is, by definition, the force per coulomb

of charge placed at the point, it follows that the force F on a charge of 1.6×10^{-19} C at a point where the electric field is 3.6×10^{10} N C^{-1} is

$$F = (3.6 \times 10^{10}) \times (1.6 \times 10^{-19}) \ \frac{\text{N}}{\text{C}} \ \text{C}$$

i.e. $$F = 5.76 \times 10^{-9} \text{ N}$$

Question 8

The sort of force we provide via our leg muscles in pushing the car is probably about the same as the force it takes to lift up say, a couple of stones of potatoes, again using our leg muscles. Now the weight of an object of mass 2×14 lb. $= 28$ lb. $= 28/2.2$ kg is 13×9.8 N $\approx 10^2$ N (the acceleration due to gravity being 9.8 m s^{-2}). Since 1 mile $\approx$ 1 500 m the energy supplied by the pusher is 'force $\times$ distance' $\approx 10^2 \times 1.5 \times 10^3 \approx 10^5$ N m, or 10^5 J.

Question 9

The energy required is by definition, the electrical potential energy of the -1.6×10^{-19} C charge when 2.5×10^{-10} m from the $+1.6 \times 10^{-19}$ C charge. From equation 17 the potential energy of a charge Q_1 when a distance R from Q_2 is given by

$$\text{P.E.} = Q_1 \, Q_2 / 4\pi\epsilon_0 R.$$

Here
$$Q_1 = -1.6 \times 10^{-19} \text{ C}$$
$$Q_2 = +1.6 \times 10^{-19} \text{ C}$$
$$R = 2.5 \times 10^{-10} \text{ m}$$
$$1/4\pi\epsilon_0 = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\therefore \ \text{P.E.} = -\frac{9 \times 10^9 \times 1.6 \times 1.6}{10^{19} \times 10^{19} \times 2.5 \times 10^{-10}} \ \frac{\text{N m}^2}{\text{C}^2} \ \frac{\text{C}^2}{\text{m}}$$

$$\text{P.E.} = -9.2 \times 10^{-19} \text{ J}$$

The minus sign indicates that energy is transferred from the electric field to the pusher in this case; the positive charge pulls in the negative one.

Question 10

The energy transferred as 1 C moves through a potential difference of 250 V, i.e. 250 J C^{-1}, is 250 J. (This follows from the definition of potential difference as the energy transferred per coulomb.) But a current of 4 A means that 4 C of charge pass through the force per second. Therefore the total thermal energy appearing in the fire per second is 250 J C^{-1} $\times$ 4 C s^{-1} $= 10^3$ J s^{-1}.

Question 11

Equation 20 shows that

$$\text{Power} = V \times I$$

i.e. $$I = \frac{\text{Power}}{V}$$

In this problem the power $= 150$ W $= 150$ J s^{-1}, and $V = 250$ V $= 250$ J C^{-1},

$$\therefore\ I = \frac{150}{250}\ \frac{\text{J s}^{-1}}{\text{J C}^{-1}}$$

i.e. $$I = 0.6 \text{ C s}^{-1} = 0.6 \text{ A}.$$

Question 12

The kinetic energy of an object of mass m moving at a speed v is $\frac{1}{2}mv^2$.
Substituting $m = 10^3$ kg and $v = 50$ m s^{-1} gives

$$\text{K.E.} = \tfrac{1}{2} \times 10^3 \times (50)^2 \ \ \text{kg}\ \frac{\text{m}^2}{\text{s}^{-2}}$$

$$= 1.25 \times 10^6 \text{ N m}$$

$$= 1.25 \times 10^6 \text{ J}$$

As the car took 20 s to obtain the K.E. its average power (the rate at which it gained K.E.) was $125 \times 10^4/20 = 6.25 \times 10^4$ J s^{-1} $= 6.25 \times 10^4$ W

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Notes

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